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(1) Find the derivative of the following using First Principle (a)  $y = \sin(3/x)$ .

(a)  $y = \sin\left(\frac{3}{x}\right)$  ;  $y = \sin 3x^{-1}$

$\Delta y = \sin 3(x+\Delta x)^{-1}$

$\Delta y + y = \sin 3(x+\Delta x)^{-1}$

$\Delta y = \sin 3(x+\Delta x)^{-1} - y$  where  $y = \sin 3x^{-1}$

$\Delta y = \sin 3(x+\Delta x)^{-1} - \sin 3x^{-1}$

let  $3(x+\Delta x)^{-1} = A$  and  $3x^{-1} = B$

$\therefore \Delta y = \sin A - \sin B$

and remember  $\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$

$\therefore \Delta y = 2 \cos\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$

$\frac{A+B}{2} = \frac{3(x+\Delta x)^{-1} + 3x^{-1}}{2}$  ;  $\frac{A-B}{2} = \frac{3(x+\Delta x)^{-1} - 3x^{-1}}{2}$

$\frac{A+B}{2} = \left(\frac{3}{(x+\Delta x)^2} + \frac{3}{x^2}\right) \frac{1}{2}$  ;  $\frac{A-B}{2} = \left(\frac{3}{(x+\Delta x)^2} - \frac{3}{x^2}\right) \frac{1}{2}$

$\frac{A+B}{2} = \left(\frac{3x^2 + 3(x+\Delta x)^2}{x^2(x+\Delta x)^2}\right) \frac{1}{2}$  ;  $\frac{A-B}{2} = \left(\frac{3x^2 - 3(x+\Delta x)^2}{x^2(x+\Delta x)^2}\right) \frac{1}{2}$

$\frac{A+B}{2} = \left(\frac{3x^2 + 3x^2 + 6x\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}\right) \frac{1}{2}$  ;  $\frac{A-B}{2} = \left(\frac{3x^2 - 3x^2 - 6x\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}\right) \frac{1}{2}$

$\frac{A+B}{2} = \left(\frac{6x^2 + 6x\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}\right) \frac{1}{2}$  ;  $\frac{A-B}{2} = \left(\frac{-6x\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}\right) \frac{1}{2}$

$\Delta y = 2 \cos\left(\frac{6x^2 + 6x\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}\right) \cdot \sin\left(\frac{-6x\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}\right)$

Divide through by  $\Delta x$

$\frac{\Delta y}{\Delta x} = 2 \cos\left(\frac{6x^2 + 6x\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}\right) \cdot \sin\left(\frac{-6x\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}\right)$

$$\frac{\Delta y}{\Delta x} = \frac{2 \cos \left( \frac{6x^2 + 6x\Delta x + 3\Delta x^2}{2x^2(x+\Delta x)^2} \right) \cdot \sin \left( \frac{-6x\Delta x - 3\Delta x^2}{2x^2(x+\Delta x)^2} \right)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 2 \cos \left( \frac{6x^2 + 6x\Delta x + 3\Delta x^2}{2x^2(x+\Delta x)^2} \right) \cdot \sin \left( \frac{-6x\Delta x - 3\Delta x^2}{2x^2(x+\Delta x)^2} \right)$$

$$\frac{\Delta y}{\Delta x} = 2 \cos \left( \frac{6x^2}{2x^2(x+\Delta x)^2} + \frac{6x\Delta x}{2x^2(x+\Delta x)^2} + \frac{3\Delta x^2}{2x^2(x+\Delta x)^2} \right) \cdot \sin \left( \frac{-6x\Delta x}{2x^2(x+\Delta x)^2} - \frac{3\Delta x^2}{2x^2(x+\Delta x)^2} \right) \div \Delta x$$

$$\frac{\Delta y}{\Delta x} = 2 \cos \left( \frac{3x^2}{(x+\Delta x)^2} + \frac{3\Delta x}{x(x+\Delta x)^2} + \frac{3\Delta x^2}{2x^2(x+\Delta x)^2} \right) \cdot \sin \left( \frac{-3\Delta x}{x(x+\Delta x)^2} - \frac{3\Delta x^2}{2x^2(x+\Delta x)^2} \right) \div \Delta x$$

$$\frac{\Delta y}{\Delta x} = 2 \cos \left( \frac{3}{(x+\Delta x)^2} + \frac{3\Delta x}{x(x+\Delta x)^2} + \frac{3\Delta x^2}{2x^2(x+\Delta x)^2} \right) \cdot \sin \left( \frac{-3\Delta x}{x(x+\Delta x)^2} - \frac{3\Delta x^2}{2x^2(x+\Delta x)^2} \right) \div \Delta x$$

multiply through by  $\frac{1}{2}$

$$\frac{\Delta y}{\Delta x} \cdot \frac{1}{2} = \cos \left( \frac{3}{(x+\Delta x)^2} + \frac{3\Delta x}{x(x+\Delta x)^2} + \frac{3\Delta x^2}{2x^2(x+\Delta x)^2} \right) \cdot \sin \left( \frac{-3\Delta x}{x(x+\Delta x)^2} - \frac{3\Delta x^2}{2x^2(x+\Delta x)^2} \right)$$

$$\frac{\Delta y}{\Delta x} = \cos \left( \frac{3}{(x+\Delta x)^2} + \frac{3\Delta x}{x(x+\Delta x)^2} + \frac{3\Delta x^2}{2x^2(x+\Delta x)^2} \right) \cdot \sin \left( \frac{-3\Delta x}{x(x+\Delta x)^2} - \frac{3\Delta x^2}{2x^2(x+\Delta x)^2} \right)$$

$$\lim_{\Delta x \rightarrow 0} \frac{dy}{dx} = \cos \left( \frac{3}{x^2} + \frac{0}{0} + \frac{0}{0} \right) \cdot \sin \left( \frac{0}{0} - \frac{0}{0} \right)$$

$$\frac{dy}{dx} = \cos \frac{3}{x^2}$$

(b)  $y = \psi/x^3$

Solution

$$y = \psi(x)^{-3}$$

$$\Delta y = \psi(x+\Delta x)^{-3} - \psi(x)^{-3}$$

$$\Delta y = \frac{\psi(x+\Delta x)^{-3} - \psi(x)^{-3}}{\Delta x}$$

$$\Delta y + y = \frac{\phi}{(x+\Delta x)^3}$$

$$\Delta y = \frac{\phi}{(x+\Delta x)^3} - y \quad \text{where } y = \frac{\phi}{x^3}$$

$$\Delta y = \frac{\phi}{(x+\Delta x)^3} - \frac{\phi}{x^3}$$

$$\Delta y = \frac{\phi x^3 - \phi (x+\Delta x)^3}{x^3 (x+\Delta x)^3}$$

$$\Delta y = \frac{\phi x^3 - \phi (x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3)}{x^3 (x+\Delta x)^3}$$

$$\Delta y = \frac{\phi x^3 - \phi x^3 - 12x^2 \Delta x - 12x \Delta x^2 - \phi \Delta x^3}{x^3 (x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3)}$$

$$\Delta y = \frac{-12x^2 \Delta x - 12x \Delta x^2 - \phi \Delta x^3}{x^3 (x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3)}$$

Let  $x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3 = t$

$$\therefore \Delta y = \frac{-12x^2 \Delta x - 12x \Delta x^2 - \phi \Delta x^3}{x^3 t}$$

$$\therefore \Delta y = \frac{-12x^2 \Delta x}{x^3 t} - \frac{12x \Delta x^2}{x^3 t} - \frac{\phi \Delta x^3}{x^3 t}$$

$$\Delta y = \frac{-12 \Delta x}{x t} - \frac{12 \Delta x^2}{x^2 t} - \frac{\phi \Delta x^3}{x^3 t}$$

Divide through by  $\Delta x$

$$\frac{\Delta y}{\Delta x} = \frac{-12 \Delta x}{x t \Delta x} - \frac{12 \Delta x^2}{x^2 t \Delta x} - \frac{\phi \Delta x^3}{x^3 t \Delta x}$$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{-12}{x(x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3)} - \frac{12}{x^2(x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3)} - \frac{\phi \Delta x^2}{x^3(x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3)}$$

$$\frac{-12}{x^2(x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3)} - \frac{\phi \Delta x^2}{x^3(x^3 + 3x^2 \Delta x + 3x \Delta x^2 + \Delta x^3)}$$

lim  $\Delta x \rightarrow 0 \quad \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-12}{x(x^3 + 0 + 0 + 0)} - 0 - 0$$

$$\frac{dy}{dx} = \frac{-12}{x^4}$$

$$\therefore \frac{dy}{dx} = -12x^{-4}$$

(2) Find the integral of the following  
 (a)  $\frac{dx}{x^2+6}$

solution.  
 $\int \frac{1}{x^2+6} dx = \int \frac{dx}{x^2+6}$

$x = 6 \tan \theta$   
 $dx = 6 \sec^2 \theta, \quad dx = 6 \sec^2 \theta d\theta$

$x^2+6 = 6 \tan^2 \theta + 6 = 6(\tan^2 \theta + 1)$

factoring after substitution.  
 $\int \frac{6 \sec^2 \theta d\theta}{36 \sec^4 \theta} = \int \frac{d\theta}{6} = \frac{1}{6} \int d\theta = \frac{1}{6} (\theta) + C$

but  $\theta = \tan^{-1} \frac{x}{6}$

$\therefore \frac{1}{6} \tan^{-1} \frac{x}{6} + C$  is the integral of  $\frac{dx}{(x^2+6)}$

(b)  $\frac{dx}{x^2+13}$

solution.

$\int \frac{dx}{x^2+13} = \int \frac{1}{x^2+13} dx$

$x = \sqrt{13} \tan \theta \quad \frac{dx}{d\theta} = \sqrt{13} \sec^2 \theta \quad dx = d\theta \sqrt{13} \sec^2 \theta$

$x^2+13 = 13 \tan^2 \theta + 13 = 13(\tan^2 \theta + 1)$

by factorization.

substitute.

$\int \frac{dx}{x^2+13} = \int \frac{d\theta \sqrt{13} \sec^2 \theta}{13 \sec^2 \theta} = \int \frac{d\theta}{\sqrt{13}} = \frac{1}{\sqrt{13}} \int d\theta$

$= \frac{1}{\sqrt{13}} (\theta) + C = \frac{1}{\sqrt{13}} (\theta) + C$

but  $\theta = \tan^{-1} \frac{x}{\sqrt{13}}$

$\frac{1}{\sqrt{13}} \tan^{-1} \frac{x}{\sqrt{13}} + C$