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1) $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

2) $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

3) $\int (\tan x)^6 \sec^2 x\,dx$

Solution.

1) $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

let $u = 4x^2 - 1$

$\left(\frac{u-1}{4}\right)^{1/2} = x$

$x = \left(\frac{u-1}{4}\right)^{1/2} \Rightarrow \frac{(u-1)^{1/2}}{2}$

$\frac{dx}{du} = \frac{1}{4(u-1)^{1/2}}$

$dx = \frac{du}{4(u-1)^{1/2}}$

$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \int \frac{2(u-1)^{1/2}}{2} \cdot \frac{1}{u^{1/2}}$

$= \int \frac{2(u-1)^{1/2}}{2} \cdot \frac{1}{u^{1/2}} \cdot \frac{du}{4(u-1)^{1/2}}$

$= \frac{1}{4} \cdot \int u^{-1/2}\,du$

$= \frac{1}{4}\left(\frac{u^{1/2}}{1/2}\right) + c$

$= \frac{2}{4}\left((4x^2-1)^{1/2}\right) + c$

$= \int \frac{2x}{\sqrt{4x^2-1}} \cdot dx \Rightarrow \frac{1}{2}\left(\sqrt{4x^2-1}\right) + c$

2) $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

let $u = \sin^{-1}x$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

$dx = du\,(\sqrt{1-x^2})$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int \frac{u}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2}\,du$

$= \int u\,du$

$= \frac{u^2}{2} + C$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{(\sin^{-1}x)^2}{2} + C$

3) $\int (4\tan x)^6 \sec^2 x\,dx$

$u = 4\tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

$\int u^6 \cdot \sec^2 x \cdot \frac{du}{\sec^2 x}$

$= \frac{u^7}{7} + C$

$\int (\tan x)^6 \sec^2 x\,dx = \frac{(\tan x)^7}{7} + C$