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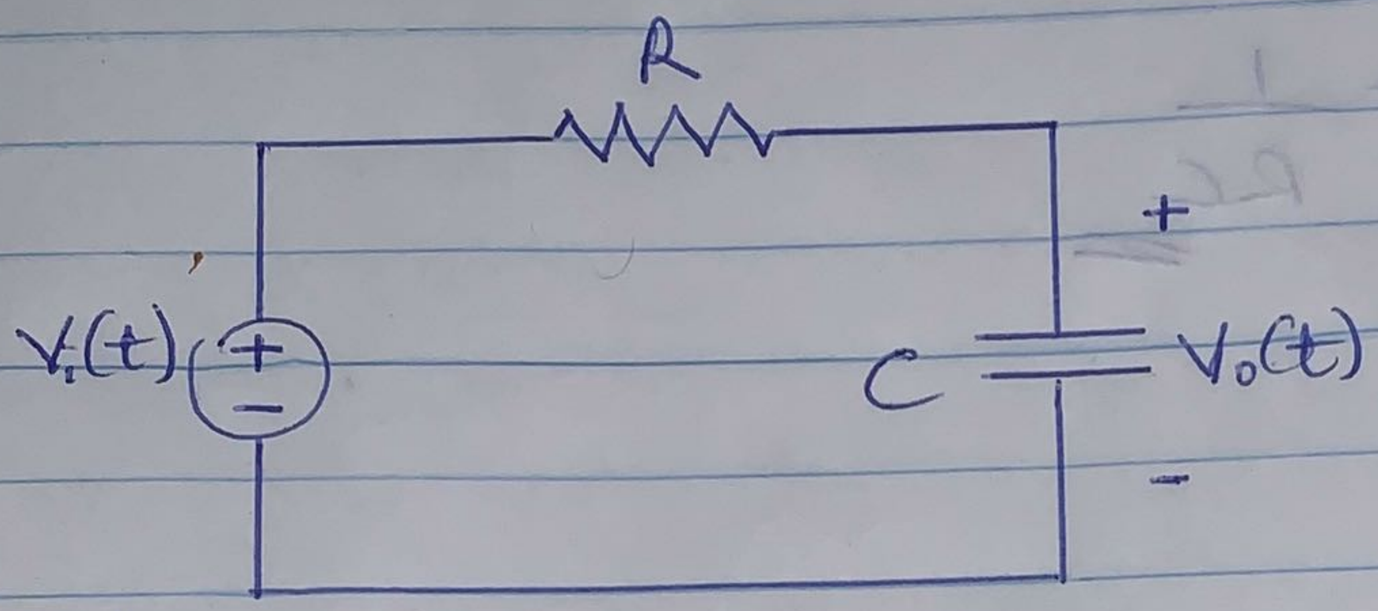
16/ENG07/018	MECHATRONICS ENGINEERING

EEE322 - CIRCUIT THEORY

(1) Determine the type of filter shown below, and show that its cut off frequency is $\omega_c = 1/RC$



Solution

Converting to frequency domain

$R = R$

$C = 1/j\omega C$

$H(\omega) = V_o/V_i$

using KVL (Series Connection)

$$H(\omega) = \frac{V_o}{V_i} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{\frac{j\omega C}{Rj\omega C + 1}} = \frac{1}{j\omega C} \times \frac{j\omega C}{Rj\omega C + 1}$$

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

when $H(0) = 1$, $H(\infty) = 0$

(i) Hence, the circuit is a Low Pass filter

By setting the magnitude of $H(\omega)$ equal to $\frac{1}{\sqrt{2}}$ to obtain out of frequency.

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$1 = 1$$

$$\sqrt{1 + \omega_c^2 R^2 C^2} = \sqrt{2}$$

$$1 + \omega_c^2 R^2 C^2 = 2$$

$$\omega_c^2 R^2 C^2 = 2 - 1$$

$$\omega^2 c R^2 C^2 = 1$$

$$(\omega_c RC) = \sqrt{1}$$

$$\omega_c RC = 1$$

$$\omega_c = \frac{1}{RC}$$