$y = \sin\left(\frac{3}{x^2}\right)$

$y + \Delta y = \sin 3\left(\frac{1}{x^2}\right)$

$y + \Delta y = \sin 3\left(\frac{1}{(x+\Delta x)^2}\right)$

$\Delta y = \sin 3\left(\frac{1}{(x^2+2\Delta x+\Delta x^2)}\right) - y$

$\Delta y = \sin 3\left(\frac{1}{(x^2+2\Delta x+\Delta x^2)}\right) - \sin 3\left(\frac{1}{x^2}\right)$

$\Delta y = \sin\frac{3}{(x^2+2\Delta x+\Delta x^2)} - \sin\left(\frac{3}{x^2}\right)$

$\Delta y = \sin 3\left(\frac{1}{(x^2+2\Delta x+\Delta x^2)}\right) - \sin 3\left(\frac{1}{x^2}\right)$

$\Delta y = \sin 3(x^2) - \sin 3(x^2+2\Delta x+\Delta x^2)$

$(x^2+2\Delta x+\Delta x^2)(x^2)$

$\Delta y = \sin 3x^2 - \sin 6\Delta x - \sin 3\Delta x^2$

$(x^2+2\Delta x+\Delta x^2)(x^2)$

$\Delta y = -\sin 6\Delta x - \sin 3\Delta x^2$

$(x^2+2\Delta x+\Delta x^2)(x^2)$

$\Delta y = -\sin 6\Delta x - \sin 3\Delta x^2$

$(x^4+2x^2\Delta x+x^2\Delta x^2)$

$\frac{\Delta y}{\Delta x} = -\frac{\sin 6 - \sin 3\Delta x}{(x^4+2x^2\Delta x+x^2\Delta x^2)}$

$\frac{\Delta y}{\Delta x} \xrightarrow{\lim} \frac{dy}{dx} \quad \frac{dy}{dx} = -\frac{\cos 6}{(x^4)}$

$\frac{dy}{dx} = -\cos\left(\frac{6}{x^4}\right)$

(b) $y = \frac{4}{x^3}$

$y = 4\left(\frac{1}{x^3}\right)$

$y + \Delta y = 4\left[\frac{1}{(x+\Delta x)^3}\right]$

$\Delta y = 4\left[\frac{1}{(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}\right] - y$

$\Delta y = 4\left[\frac{1}{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3}\right] - \frac{4}{x^3}$

$\Delta y = 4\left[\frac{x^3}{(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}\right] - 4\left(\frac{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3}{x^3}\right)$

$\Delta y = \frac{4x^3 - 4x^3 - 12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)(x^3)}$

$\Delta y = \frac{-12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)(x^3)}$

$\frac{\Delta y}{\Delta x} = \frac{-12x^2 - 12x\Delta x - 4\Delta x^2}{(x^9 + 3x^6\Delta x + 3x^3\Delta x^2 + x^3\Delta x^3)}$

$\Delta y \xrightarrow{\lim} \frac{dy}{dx} = \frac{-12x^2}{(x^9)}$

$\Delta x \to 0 \quad dx$

$\frac{dy}{dx} = -12\left(\frac{x^2}{x^9}\right)$

$\frac{dy}{dx} = -12/x^7$

(a) $\frac{dx}{(x^2+36)}$

$\int \left(\frac{1}{x^2+36}\right) dx$.

let $u = x^2 + 36$.

$\frac{du}{dx} = 2x$.

$du = 2x\,dx$.

$dx = \frac{du}{2x}$.

$\int \left(\frac{1}{u}\right) \frac{du}{2x}$.

$\int \frac{2x}{u}\, du$.

$2x \int \frac{1}{u}\, du$.

$2x \ln u + c$.

$2x \ln(x^2+36) + c$.

$$\frac{dx}{(x^2+13)}$$

$$\int \frac{1}{(x^2+13)}\, dx.$$

Let $u = x^2 + 13$.

$$\frac{du}{dx} = 2x.$$

$$du = 2x\, dx.$$

$$dx = \frac{du}{2x}.$$

$$\int \left(\frac{1}{u}\right) \frac{du}{2x}.$$

$$\int \frac{2x}{u}\, du.$$

$$2x \int \frac{1}{u}\, du.$$

$$2x \ln u + c.$$

$$2x \ln (x^2+13) + c.$$