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MECHATRONICS ENGINEERING
19/ENG05/043
MAT 104 ASSIGNMENT

1)

a) $y = \sin\left(3/x^2\right)$

$y + \Delta y = \sin\left(3/(x+\Delta x)^2\right)$

$\Delta y = \sin\left[\frac{3}{(x+\Delta x)^2}\right] - y$

$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\Delta y = 2\cos\frac{\frac{3}{(x+\Delta x)^2} + \frac{3}{x^2}}{2}\sin\frac{\frac{3}{(x+\Delta x)^2} - \frac{3}{x^2}}{2}$

$\frac{\Delta y}{\Delta x} = \frac{2\cos\frac{3x^2+3(x+\Delta x)^2}{2x^2(x+\Delta x)^2}\sin\frac{3x^2-3(x+\Delta x)^2}{2x^2(x+\Delta x)^2}}{\Delta x}$

$\frac{\Delta y}{\Delta x} = \left[2\cos\frac{3x^2+3(x+\Delta x)^2}{2x^2(x+\Delta x)^2}\sin\frac{-6x\Delta x-3(\Delta x)^2}{2x^2(x+\Delta x)^2}\right] \times \frac{1}{\Delta x}$

$\frac{\Delta y}{\Delta x} = 2\cos\frac{3x^2+3(x+\Delta x^2)}{2x^2(x+\Delta x)^2}\sin\frac{-6x\Delta x-3\Delta x^2}{2x^2(x+\Delta x)^2} \times \frac{-6x-3\Delta x}{2x^2(x+\Delta x)^2}$

$\frac{-6x\Delta x-3\Delta x^2}{2x^2\cdot(x+\Delta x)^2}$

$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = 2\cos\frac{3x^2+3(x+0)^2}{2x^2(x+0)^2} \times \frac{-6x-3(0)}{2x^2(x+0)^2}$

$\therefore \frac{dy}{dx} = \frac{-6\cancel{x}}{\cancel{x^2}}\ \frac{-6x}{x^4}\ \cos\frac{{}^3\cancel{6}x^2}{{}_1\cancel{2}x^4}$

$\therefore \frac{dy}{dx} = \frac{-6}{x^3}\cos\frac{3}{x^2}$

b) $y = \frac{4}{x^3}$

$y + \Delta y = \frac{4}{(x+\Delta x)^3}$

$$\Delta y = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4x^3 - 4(x+\Delta x)^3}{x^3(x+\Delta x)^3}$$

$$\Delta y = \frac{4x^3 - 4[(x+\Delta x)(x^2 + 2x\Delta x + (\Delta x)^2)]}{x^3(x+\Delta x)^3}$$

$$\Delta y = \frac{\cancel{4x^3}\ \cancel{4x^3} - 8x^2\Delta x - 4x\Delta x^2 - 4x^2\Delta x - 8x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{-8x^2\Delta x - 4x\Delta x^2 - 4x^2\Delta x - 8x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3} \times \frac{1}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cancel{\Delta x}(-8x^2 - 4x\Delta x - 4x^2 - 8x\Delta x - 4\Delta x^2)}{x^3(x+\Delta x)^3} \times \frac{1}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-8x - 4x(0) - 4x^2 - 8x(0) - 4(0)^2}{x^3(x+0)^3}$$

$$\frac{dy}{dx} = \frac{-8x^2 - 4x^2}{x^3(x^3)} = \frac{-8x^2 - 4x^2}{x^6} = \frac{-12x^2}{x^6}$$

$$\frac{dy}{dx} = \frac{-12}{x^4}$$

2) a) $\displaystyle\int \frac{dx}{x^2+36}$

Let $x = a\tan\theta$.

where $a = 6$

$x = 6\tan\theta$

$\frac{dx}{d\theta} = 6\sec^2\theta \quad \therefore dx = 6\sec^2\theta\, d\theta$

$\therefore x^2 + 36 = (6\tan\theta)^2 + 36$

$= 36\tan^2\theta + 36$

$= 36(\tan^2\theta + 1)$

$36 + x^2 = 36(1 + \tan^2\theta)$.

Recall that $1 + \tan^2\theta = \sec^2\theta$.

$= 36 + x^2 = 36\sec^2\theta$

$$\int \frac{dx}{x^2+36} = \int \frac{6\sec^2\theta\, d\theta}{36\sec^2\theta}$$

$$= \frac{1}{6}\int d\theta = \frac{1}{6}[\theta] + c$$

$$= \frac{1}{6}\tan^{-1}\frac{x}{6} + C$$

b) $\int \frac{1}{x^2+13}\,dx = \int \frac{1}{x^2+(\sqrt{13})^2}$

let $x = a\tan\theta$.

where $a = \sqrt{13}$

$x = \sqrt{13}\tan\theta$.

$\frac{dx}{d\theta} = \sqrt{13}\sec^2\theta$.

$dx = \sqrt{13}\sec^2\theta\, d\theta$.

$x^2+13 = \cancel{(\sqrt{13}\sec^2\theta)}\ (\sqrt{13}\tan\theta)^2 + 13$.

$= 13\tan^2\theta + 13$

$x^2+13 = 13(\tan^2\theta + 1)$.

Recall than $1 + \tan^2\theta = \sec^2\theta$.

$x^2+13 = 13\sec^2\theta$.

$$\int \frac{1}{x^2+13}\,dx = \int \frac{\sqrt{13}\,\cancel{\sec^2\theta}\, d\theta}{13\,\cancel{\sec^2\theta}}$$

$$= \frac{\sqrt{13}}{13}\int d\theta$$

$$= \frac{\sqrt{13}}{13}[\theta] + C$$

$$= \frac{\sqrt{13}}{13}\tan^{-1}\frac{x}{\sqrt{13}} + C$$