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D. CYL(AM)

Find dy/dx of $y = \sin 3/x^2$ using first principle

If $y = \sin\left(3/x^2\right) = f(x)$ let $h = \Delta x$

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \sin\frac{3}{(x+h)^2} - \sin\frac{3}{x^2}$$

if $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$$= \lim_{h \to 0} \frac{2\cos\left[\frac{\frac{3}{(x+h)^2} + \frac{3}{x^2}}{2}\right] \cdot \sin\left[\frac{\frac{3}{(x+h)^2} + 3/x^2}{2}\right]}{h}$$

$$\lim_{h} \frac{2\cos\left(\frac{\frac{3}{(x+h)^2} + 3/x^2}{2}\right) \cdot \lim \sin\left[\frac{\frac{3x^2 - 3(x+h)^2}{x^2(x+h)^2}}{2}\right]}{h}$$

$$2\cos\frac{\left(3/x^2 + 3/x^2\right)}{2} \lim_{h \to 0} \sin\left[\frac{\frac{3x^2 - 3x^2 - 6xh - 3h^2}{x^2(x+h)^2}}{2}\right]$$

$$2\cos 3/x^2 \times \lim \sin \frac{\frac{-6xh - 3h^2}{2x^2(x+h)^2}}{h}$$

Since $\lim_{\theta \to 0} \frac{\sin \theta h}{\theta} = \theta$

$$2\cos\left(\frac{3}{x^2}\right) \times \lim \frac{-6x-3h}{2x^2(x+h)^2}$$

$$2\cos\left(\frac{3}{x^2}\right) \times \frac{-6x}{2x^4}$$

$$= \frac{-6x}{x^4} \cos \frac{3}{x^2}$$

$$= \frac{-6}{x^3} \cos\left(\frac{3}{x^2}\right)$$

(b) $y = \frac{4}{x^3}$

$$y + \Delta y = \frac{4}{(x + \Delta x)^3}$$

$$\Delta y = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3} \div \Delta x$$

$$\frac{\Delta y}{\Delta x} = \frac{4x^3 - 4(x+\Delta x)^3}{x^3(x+\Delta x)^3} \div \Delta x$$

$$\frac{\Delta y}{\Delta x} = \frac{4x^3 - 4(x^3 + 3\Delta x^2 \Delta x + 3x\Delta x^2 + \Delta x^3)}{x^3(x + \Delta x)^3} \div \Delta x$$

$$\frac{\Delta y}{\Delta x} = \frac{4x^3 - 4x^3 - 12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3} \div \Delta x$$

$$\frac{-12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3} \div \Delta x$$

$$= \frac{-12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(x+\Delta x)^3} \times \frac{1}{\Delta x}$$

$$= \frac{-12x^2 - 12x\Delta x - 4\Delta x^2}{x^3(x+\Delta x)^3}$$

$$\lim_{\Delta x \to 0} = \frac{-12x - 12x(0) - 4(0)^2}{x^3(x+0)^3}$$

$$= \frac{-12x^2}{x^6}$$

$$= \frac{-12}{x^4}$$

2 $\int \frac{dx}{x^2+36}$

$$= \int \frac{dx}{x^2+6^2}$$

$x = 6\tan\theta$

$\frac{dx}{d\theta} = 6\sec^2\theta$

$dx = 6\sec^2\theta\, d\theta$

$x^2 + 6x = 6^2\tan^2\theta + 6^2$

$= 6^2(\tan^2\theta + 1)$

$= 36(\tan^2\theta + 1$

$= 36\sec^2\theta$

$\frac{6\sec^2\theta\, d\theta}{36\sec^2\theta}$ $\int \frac{d\theta}{6}$

$\frac{1}{6}\int d\theta$

$\theta = \tan^{-1}\frac{x}{6}$

$\frac{1}{6}[\theta] + c$

$= \frac{1}{6}\tan^{-1}\frac{x}{6} + c$

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6 $\int \frac{dx}{x^2+13}$

$x = \sqrt{13} \tan\theta \longrightarrow \theta = \frac{x}{\sqrt{13}} \tan^{-1}$

$\frac{dx}{d\theta} = \sqrt{13} \sec^2\theta$

$dx = \sqrt{13} \sec^2\theta \, d\theta$

$x^2 + (\sqrt{13})^2 = (\sqrt{13})^2 \tan^2\theta (\sqrt{13})^2$

$= (\sqrt{13})^2 (\tan^2\theta + 1)$

$= (\sqrt{13})^2 \sec^2\theta = 13 \sec^2\theta$

$= \frac{\sqrt{13} \sec^2\theta \, d\theta}{13 \sec^2\theta}$

$= \int \frac{\sqrt{13}}{13} d\theta$

$= \frac{\sqrt{13}}{13} \int d\theta$

$= \frac{\sqrt{13}}{13} [\theta] + C$

$= \frac{\sqrt{13}}{13} + \frac{x}{13} \tan^{-1} + C$