

3. Given that  $\mathbf{r} = (\sin 3t)\mathbf{i} + \phi e^{3t}\mathbf{j} + 7t^3\mathbf{k}$ . Find the integral for  $R$  with respect to  $t$ .

Solution.

$$\mathbf{r} = \phi \sin 3t \mathbf{i} + \phi e^{3t} \mathbf{j} + 7t^3 \mathbf{k}$$

$$d\mathbf{r} = \phi \cos 3t \mathbf{i} + 3\phi e^{3t} \mathbf{j} + 21t^2 \mathbf{k}$$

$$R = \phi \int \sin 3t \mathbf{i} + \phi \int e^{3t} \mathbf{j} + \int 7t^3 \mathbf{k}$$

$$R = \frac{\phi}{3} (-\cos 3t) \mathbf{i} + \frac{\phi e^{3t}}{3} \mathbf{j} + \frac{7t^4}{4} \mathbf{k}$$

$$\int R = \frac{-\phi \cos(3t)}{3} \mathbf{i} + \frac{\phi e^{3t}}{3} \mathbf{j} + \frac{7t^4}{4} \mathbf{k}$$

4. If  $A = 7\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ ,  $B = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ ,  $C = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$  find  $(A+C)(B-A)$   
 $(A+C)(B-A) = [(7\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) + (\mathbf{i} + \mathbf{j} - 4\mathbf{k})] \cdot [2\mathbf{i} + \mathbf{j} + 4\mathbf{k}] - (7\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) \cdot (2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$   
 $= [8\mathbf{i} + 3\mathbf{j} - 8\mathbf{k}] \cdot [2\mathbf{i} + \mathbf{j} + 4\mathbf{k}] - [14 + 2 - 16]$   
 $= 16 - 3 - 32$   
 $= -19$

5. Find the unit vector tangent to the space curve  $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + t^4\mathbf{k}$  at point where  $t=1$ .

$$\mathbf{T} = \frac{d\mathbf{r}}{dt} \Big/ \left| \frac{d\mathbf{r}}{dt} \right|$$

$$\mathbf{r} = t^2\mathbf{i} + t^3\mathbf{j} + t^4\mathbf{k}$$

$$\mathbf{r} = t^2\mathbf{i} + t^3\mathbf{j} + t^4\mathbf{k}$$

$$\frac{d\mathbf{r}}{dt} = 2t\mathbf{i} + 3t^2\mathbf{j} + 4t^3\mathbf{k}$$

$$\text{at } t=1 \quad \frac{d\mathbf{r}}{dt} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$$

$$= 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$$

$$\left| \frac{d\mathbf{r}}{dt} \right|_{t=1} = \sqrt{(2)^2 + (3)^2 + (4)^2}$$

$$= \sqrt{4 + 9 + 16}$$

$$= \sqrt{29}$$

$$= \sqrt{29} \approx 5.385$$

$$\text{Hence } \mathbf{T} = \frac{2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}}{\sqrt{29}}$$

NAME: Nkhona-Eneka Emmanuel

DEPARTMENT: Computer Science

MATHEMATICS: EUCS011067

D  $x = 7t^2$ ,  $y = 6t^2 - 4t$  and  $z = t - 5$

$$r = xi + yj + zk$$

$$r = (7t^2)i + (6t^2 - 4t)j + (t - 5)k$$

$$\text{velocity} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = 14t i + (12t - 4)j + k$$

3) If  $A = 2i + 3j - 4k$ ,  $B = 2i - 3j + k$  and  $C = 5i - 3k$  find  $A \times (B \times C)$

Solution

$$A = 2i + 3j - 4k$$

$$B = 2i - 3j + k$$

$$C = 5i - 3k$$

$$(B \times C) = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 0 & -3 \end{vmatrix}$$

$$(B \times C) = \begin{vmatrix} -3 & 1 & -j \\ 4 & -3 & 0 \end{vmatrix} + k \begin{vmatrix} 2 & -3 \\ 0 & -3 \end{vmatrix}$$

$$= -i(9 - 4) - j(-6 - 0) + k(8 - 0)$$

$$= 5i + 6j + 8k$$

$$A(B \times C) = \begin{vmatrix} i & j & k \\ 2 & 3 & -4 \\ 5 & 6 & 8 \end{vmatrix}$$

$$A(B \times C) = i \begin{vmatrix} 3 & -4 \\ 6 & 8 \end{vmatrix} - j \begin{vmatrix} 2 & -4 \\ 5 & 8 \end{vmatrix} + k \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}$$

$$A \times (B \times C) = i(16 + 24) - j(8 + 20) + k(10 - 6)$$

$$= 40i - 28j + 4k$$

$$A = i + 2j - 4k \quad B = 2i - 3j + k \quad C = 4i - 3k \quad \text{Find } A \times (B \times C)$$

Solution

$$A = i + 2j - 4k$$

$$B = 2i - 3j + k$$

$$C = 4i - 3k$$

$$(B \times C) = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 0 & -3 \end{vmatrix}$$

$$\begin{aligned} (B \times C) &= \begin{vmatrix} -3 & 1 & -j \\ 4 & -3 & i \\ 0 & 0 & -3 \end{vmatrix} + k \begin{vmatrix} 2 & -3 \\ 0 & 0 \end{vmatrix} \\ &= -i(9 - 4) - j(-6 - 0) + k(8 - 0) \\ &= 5i + 6j + 8k \end{aligned}$$

$$A(B \times C) = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 5 & 6 & 8 \end{vmatrix}$$

$$A(B \times C) = i \begin{vmatrix} 2 & -4 \\ 6 & 8 \end{vmatrix} - j \begin{vmatrix} 1 & -4 \\ 5 & 8 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix}$$

$$\begin{aligned} A \times (B \times C) &= i(16 + 24) - j(8 + 20) + k(6 - 10) \\ &= 40i - 28j + 4k \end{aligned}$$