

I. $(11-3x) / (x^2+2x-3)$

$\frac{11-3x}{(x^2+2x-3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$

$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$

$11-3x = Ax + 3A + Bx - B$

Comparing

$x: -3 = A + B$ — (1)

constant $11 = 3A - B$ — (2)

Using equation 1.

$(A = -3 - B)$ — (iii)

Subbing eqn (iii) into (2)

$11 = 3(-3 - B) - B$

$11 = -9 - 3B - B$

$11 = ~~-9~~ -9 - 4B$

$11 + 9 = -4B$

$20 = -4B$

$B = \frac{20}{-4} = \left(\frac{10}{2}\right) = -5$

$B = -5$

Subbing B into eqn (1)

$-3 = A - 5$

$-3 + 5 = A$

$A = 2$

$$I. \frac{(11-3x)}{x^2+2x-3}$$

$$11-3x$$

$$(x^2+2x-3)$$

$$= \frac{11-3x}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

$$11-3x = Ax+3A+Bx-B$$

Comparing

$$x: -3 = A+B \quad \text{--- (1)}$$

$$\text{constant } 11 = 3A - B \quad \text{--- (2)}$$

Using equation 1.

$$(A = -3 - B) \quad \text{--- (1)}$$

Subbing eqn (1) into (2)

$$11 = 3(-3 - B) - B$$

$$11 = -9 - 3B - B$$

$$11 = \cancel{-9} - 9 - 4B$$

$$11 + 9 = -4B$$

$$20 = -4B$$

$$B = \frac{20}{-4} = \left(\frac{10}{2}\right) = -5$$

$$B = -5$$

Subbing B into eqn (1)

$$-3 = A - 5$$

$$-3 + 5 = A$$

$$A = 2$$

Assignment

- 1. $(1-3x)/x^2+2x-3$
- 2. $(2x^2-9x-35)/(x+1)(x-2)(x+3)$
- 3. $1/(x^2+121)$

Solution

$$3. \frac{1}{x^2+121} dx$$

$$\frac{dx}{x^2+121} = \frac{dx}{x^2+11^2}$$

$$\text{let } x = \cancel{11 \tan \theta} \quad 11 \tan \theta$$

$$\frac{dx}{d\theta} = 11 \sec^2 \theta$$

$$dx = 11 \sec^2 \theta d\theta$$

$$x^2+11^2 = 11^2 \tan^2 \theta + 11^2$$

$$= 11^2 (\tan^2 \theta + 1) \Rightarrow \text{recall } \tan^2 \theta + 1 = \sec^2 \theta$$

$$= 11^2 \sec^2 \theta = 121 \sec^2 \theta$$

$$= \int \frac{11 \sec^2 \theta d\theta}{121 \sec^2 \theta} = \int \frac{d\theta}{11}$$

$$\frac{1}{11} [\theta] + C$$

$$= \frac{1}{11} \tan^{-1} \frac{x}{11} + C$$

$$6 + 8 = 14$$

$$8 - 6 = 2c$$

$$\frac{2}{-2} = \frac{-2c}{-2}$$

$$c = -1$$

Subbing C into eqn (ii)

$$\Rightarrow -9 = 2 + 3B - 2c$$

$$\Rightarrow -9 = 2 + 3B - 2$$

$$-9 = 3B$$

$$B = -3$$

Subbing A and B into eqn (i)

$$2 = A + (-3) + 1$$

$$2 = A - 4$$

$$A = 6$$

$$\therefore \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$= \int \left(\frac{6}{(x+1)} + \frac{(-3)}{x-2} + \frac{-1}{x+3} \right) dx$$

$$= \int \left(\frac{6}{x+1} - \frac{3}{x-2} - \frac{1}{x+3} \right) dx$$

$$= 6 \ln(x+1) - 3 \ln(x-2) - 1 \ln(x+3) + C$$