ME: OKWUOKWU BRYAN   COURSE CODE: MAT 102
MATRIC NO.: 19/ENG05/049   DEPT.: Mechatronics

$A = 5i - 7j - 6k$, $B = j + 4k$, $C = 9i - 4j + k$

$(C - A) = (9i - 4j + k) - (5i - 7j - 6k) = 9i - 5i - 4j + 7j + k + 7k = 4i + 3j + 8k$

$(A+B) = 5i - 7j - 6k + j + 4k = 5i - 6j - 2k$

$-8(A+B) = -40i + 48j + 16k$

$-8(A+B) \cdot (C - A) = (-40i + 48j + 16k) \cdot (4i + 3j + 8k)$

$= -160 + 144 + 128 = 112$

$\therefore -8(A+B) \cdot (C - A) = \underline{112}$

2) $r(t) = -3t\hat{i} + t^2\hat{j} + 4t^3\hat{k}$

$r'(t) = -3\hat{i} + 2t\hat{j} + 12t^2\hat{k}$

$|r'(t)| = \sqrt{(-3)^2 + (2t)^2 + (12t^2)^2} = \sqrt{9 + 4t^2 + 144t^4}$

$T(t) = \dfrac{r'(t)}{|r'(t)|} = \dfrac{-3\hat{i} + 2t\hat{j} + 12t^2\hat{k}}{\sqrt{9 + 4t^2 + 144t^4}}$

But $t = 1$

$T(1) = \dfrac{-3\hat{i} + 2(1)\hat{j} + 12(1)^2\hat{k}}{\sqrt{9 + 4(1)^2 + 144(1)^4}} = \dfrac{-3\hat{i} + 2\hat{j} + 12\hat{k}}{\sqrt{157}}$

$\therefore T(1) = \dfrac{-3\hat{i}}{\sqrt{157}} + \dfrac{2\hat{j}}{\sqrt{157}} + \dfrac{2\hat{k}}{\sqrt{157}}$

③ $L(t) = 8t^2\hat{i} + (t^2-4t)\hat{j} + (t+1)\hat{k}$

$L'(t) = V(t) = 16t\hat{i} + (2t-4)\hat{j} + (1)\hat{k}$

$L''(t) = a(t) = 16\hat{i} + 2\hat{j} + 0\hat{k}$

$\therefore$ acceleration $= a(t) = 16\hat{i} + 2\hat{j}$

④ $A = i + 2j - 4k \quad B = 2i - 3j + k \quad C = 4j - 3k$

$(A \times B) = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 2 & -3 & 1 \end{vmatrix} = i(2-12) - j(1+8) + k(-3-4)$

$= -10i - 9j - 7k$

$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -10 & -9 & -7 \\ 0 & 4 & -3 \end{vmatrix} = i(27+28) - j(30-0) + k(-40-0$

$= 55i - 30j - 40k$

$\therefore (A \times B) \times C = 55i - 30j - 40k$

⑤ $R(t) = 4\sin 3t\,\hat{i} + 4e^{3t}\hat{j} + 7t^3\hat{k}$

$\int_0^1 R(t)\,dt = \int_0^1 4\sin 3t\,\hat{i} + 4e^{3t}\hat{j} + 7t^3\hat{k}$

$= \left[\frac{-4}{3}\cos 3t\,\hat{i} + \frac{4e^{3t}}{3}\hat{j} + \frac{7t^3}{3}\hat{k}\right]_0^1$

$= \left[\frac{-4}{3}\cos 3(1)\,\hat{i} + \frac{4}{3}e^{3(1)}\hat{j} + \frac{7(1)^3}{3}\hat{k}\right]$

$- \left[\frac{-4}{3}\cos 3(0)\,\hat{i} + \frac{4}{3}e^{3(0)}\hat{j} + \frac{7(0)^3}{3}\hat{k}\right]$

$$\int_0^1 B(t)\,dt = \left[-\frac{4}{3}\cos 3 + \frac{4}{3}\cos 0\right]\hat{i} + \left[\frac{4}{3}e^{3} - \frac{4}{3}\right]\hat{j} + \left[\frac{7}{3} - 0\right]\hat{k}$$

$$\int_0^1 B(t)\,dt = \left[-1.31999 + \frac{4}{3}\right]\hat{i} + \left[26.7807 - \frac{4}{3}\right]\hat{j} + \left[2\tfrac{1}{3}\right]\hat{k}$$

$$\int_0^1 B(t)\,dt = 0.01334\,\hat{i} + 25.4474\,\hat{j} + 2.333\,\hat{k}$$