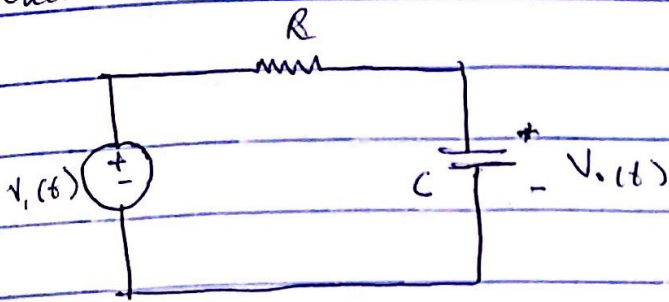


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HW ENG of LOSS
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Determine the type of filter shown below and show that it is a ~~cut off~~ ^{cut off} frequency is $\omega_c = \frac{1}{RC}$

Solution.

Converting to frequency domain

$$R - R$$

$$C = \frac{1}{j\omega C}$$

$$H(\omega) = \frac{V_o}{V_i}$$

Using Kirchhoff voltage law (series connection)

$$H(\omega) = \frac{V_o}{V_i} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1/j\omega C}{\frac{Rj\omega C + 1}{j\omega C}}$$

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

when $H(\omega_c) = \frac{1}{\sqrt{2}}$, $H(\omega) = 0$

The half power frequency which is equivalent to the corner frequency on the Bode plots but in the context of filters is usually known as cut off frequency ω_c , is obtained by setting magnitude of $H(\omega)$ equal to $\frac{1}{\sqrt{2}}$

Substituting ω_c

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

Making ω_c Subject of formula

Taking square of both sides

$$\frac{1^2}{1 + \omega_c^2 R^2 C^2} = \frac{1}{2}$$

Cross multiply

$$2 \cdot 1^2 = 1 + \omega_c^2 R^2 C^2$$

$$2 = 1 + \omega_c^2 R^2 C^2$$

$$2 - 1 = \omega_c^2 R^2 C^2$$

$$1 = \omega_c^2 R^2 C^2$$

Taking square root of both sides

$$\sqrt{1} = \sqrt{\omega_c^2 R^2 C^2}$$

$$1 = \omega_c R C$$

Divide both sides with $R C$

$$\omega_c = \frac{1}{R C}$$