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MATRIC No: 19/SC09/003
DEPT: Industrial Chemistry
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1a. $y = \sin\left(\frac{3}{x^2}\right)$ & $u = 3x^{-2}$ $\therefore y = \sin u$

$\Delta y + y = \sin(u + \Delta u)$

$\Delta y = \sin(u+\Delta u) - \sin u$

recall $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$ where $A = \sin(u+\Delta u)$, $B = \sin u$

$\Delta y = 2\cos\frac{u+\Delta u+u}{2}\sin\frac{u+\Delta u-u}{2}$ $\quad \Delta y = 2\cos\frac{2u+\Delta u}{2}\sin\frac{\Delta u}{2}$

$\frac{\Delta y}{\Delta u} = \frac{2\cos\frac{2u+\Delta u}{2}\sin\frac{\Delta u}{2}}{\Delta u}$ multiply through $\frac{1}{2}$

$\frac{\Delta y}{\Delta u} = \frac{\cos\frac{u+\Delta u}{2}\sin\frac{\Delta u}{2}}{\frac{\Delta u}{2}}$ $\quad \therefore \lim_{\Delta u \to 0}\frac{\Delta y}{\Delta u}$ $\cos(u+0)$ $\frac{\sin\frac{\Delta u}{2}}{\frac{\Delta u}{2}} = 1$

$\frac{\Delta y}{\Delta u} = \cos u \times 1$ $\quad \therefore \frac{\Delta y}{\Delta u} = \cos u$

Also $u = \frac{3}{x^2}$ $= \Delta u + u = \frac{3}{(x+\Delta x)^2}$

$u = \frac{3}{x^2+2x\Delta x+\Delta x^2} - \frac{3}{x^2}$ $\quad u = \frac{3x^2 - 3x^2 - 6x\Delta x - 3\Delta x^2}{(x^2+2x\Delta x+\Delta x^2)(x^2)}$

$u = \frac{-6x\Delta x - 3\Delta x^2}{x^4+2x^3\Delta x+x^2\Delta x^2}$ $\quad \therefore \frac{\Delta u}{\Delta x} = \frac{-6x\Delta x - 3\Delta x^2}{x^4+2x^3\Delta x+x^2\Delta x^2} \times \frac{1}{\Delta x}$

$\frac{\Delta u}{\Delta x} = \frac{\Delta x(-6x-3\Delta x)}{x^4+2x^3\Delta x+x^2\Delta x^2} \times \frac{1}{\Delta x}$ $\quad \frac{\Delta u}{\Delta x} = \frac{-6x-3\Delta x}{x^4+2x^3\Delta x+x^2\Delta x^2}$

$\lim_{\Delta x \to 0}\frac{\Delta u}{\Delta x} = \frac{-6x-3(0)}{x^4+2(0)+(0)} = \frac{-6x}{x^{4}} = \frac{-6}{x^3}$

$\therefore \frac{\Delta u}{\Delta x} = -6x^{-3}$

$\therefore \frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

$= -6x^{-3} \times \cos u$ $\quad$ recall that $u = \frac{3}{x^2}$

$\therefore \frac{dy}{dx} = -6x^{-3} \times \cos\left(\frac{3}{x^2}\right) = \frac{-6}{x^3}\cos\frac{3}{x^2}$

1b. $y = \frac{4}{x^3}$ $\quad y + \Delta y = \frac{4}{(x+\Delta x)^3}$

$\Delta y = \frac{4}{x^3+3x^2\Delta x+3x\Delta x^2+\Delta x^3} - \frac{4}{x^3} = \frac{\cancel{4x^3} - \cancel{4x^3} - 12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{(x^3+3x^2\Delta x+3x\Delta x^2+\Delta x^3)(x^3)}$

$\frac{\Delta y}{\Delta x} = \frac{-12x^2\Delta x - 12\Delta x^2 - 4\Delta x^3}{x^6+3x^5\Delta x+3x^4\Delta x^2+x^3\Delta x^3} \times \frac{1}{\Delta x} = \frac{\Delta x(-12x^2 - 12\Delta x - 4\Delta x^2)}{x^6+3x^5\Delta x+3x^4\Delta x^2+x^3\Delta x^3} \times \frac{1}{\Delta x}$

$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \frac{-12x^2 - 12(0) - 4(0)^2}{x^6+3(0)+3(0)+0} = \frac{-12x^2}{x^6} = \frac{-12}{x^4}$

$\therefore \frac{\Delta y}{\Delta x} = \frac{-12}{x^4}$

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2a $\int \frac{dx}{(x^2+36)}$

$\int \frac{dx}{(x^2+36)} = \int \frac{dx}{(x^2+(6)^2)}$

Recall that the integral of $\frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$\therefore \int \frac{dx}{(x^2+36)} = \int \frac{dx}{(x^2+(6)^2)}$, where $a = 6$

$= \frac{1}{6}\tan^{-1}\frac{x}{6} + c$

2b. $\frac{dx}{(x^2+13)}$

$\frac{dx}{(x^2+13)} = \int \frac{dx}{(x^2+\sqrt{13})}$

Recall that the integral of $\frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$\therefore \int \frac{dx}{(x^2+\sqrt{13})}$ where $a = \sqrt{13}$

$= \frac{1}{\sqrt{13}}\tan^{-1}\frac{x}{\sqrt{13}} + c$