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DEPARTMENT: COMPUTER SCIENCE

MATRIC NUMBER: 19/SCI01/015

ASSIGNMENT

1a. $y = \sin\left(\frac{3}{x^2}\right)$

Solution

$y = \sin\left(\frac{3}{x^2}\right) - f(x) = \sin\left(\frac{3}{x^2}\right) = f(x+\Delta x) = \sin\left(\frac{3}{(x+\Delta x)^2}\right) = \sin\left(\frac{3}{x^2+2x\Delta x+(\Delta x)^2}\right)$

$f(x+\Delta x) = \sin\left(\frac{3}{x^2+2x\Delta x+(\Delta x)^2}\right)$

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\sin\left(\frac{3}{x^2+2x\Delta x+(\Delta x)^2}\right) - \sin\left(\frac{3}{x^2}\right)}{\Delta x}$$

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{2\cos\left(\frac{\frac{3}{(x^2+2x\Delta x+(\Delta x)^2} + \frac{3}{x^2}}{2}\right)\sin\left(\frac{\frac{3}{x^2+2x\Delta x+(\Delta x)^2} - \frac{3}{x^2}}{2}\right)}{\Delta x}$$

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = 2\cos\left(\frac{6x^2+6x\Delta x+3\Delta x^2}{2x^2(x^2+2x\Delta x+(\Delta x)^2)}\right)\sin\left(\frac{-6x\Delta x - 3(\Delta x)^2}{2x^2(x^2+2x\Delta x+(\Delta x)^2)}\right)$$

$$= -2\cos\left(6x^2+6x\Delta x+3(\Delta x)^2\right)\sin\left(6x\Delta x+3(\Delta x)^2\right.$$

1b. $y = \frac{4}{x^3}$

Solution

$f(x) = \frac{4}{x^3}$

Using $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

$$f(x+\Delta x) = \frac{4}{(x+\Delta x)^3}$$

$$= \frac{4}{(x+\Delta x)(x+\Delta x)^2} = \frac{4}{(x+\Delta x)(x^2+2x\Delta x+(\Delta x)^2} = \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3}$$

$$= \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3}$$

$$f(x+\Delta x) - f(x) = \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3} - \frac{4}{x^3}$$

$$f(x+\Delta x) - f(x) = \frac{4x^3 - (4[x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3])}{x^3(x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3}$$

$$f(x+\Delta x) - f(x) = \frac{4x^3 - (4x^3+12x^2\Delta x+12x(\Delta x)^2+4(\Delta x)^3)}{x^3(x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3}$$

$$f(x+\Delta x) - f(x) = \frac{\cancel{4x^3} - \cancel{4x^3} - 12x^2\Delta x - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^6+3x^5\Delta x+3x^4(\Delta x)^2+x^3(\Delta x)^3}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{-12x^2\Delta x - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^6+3x^5\Delta x+3x^4(\Delta x)^2+x^3(\Delta x)^3} \div \Delta x$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\cancel{\Delta x}(-12x^2 - 12x\Delta x - 4(\Delta x)^2)}{x^6+3x^5\Delta x+3x^4(\Delta x)^2+x^3(\Delta x)^3} \times \frac{1}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{-12x^2 - 12x\Delta x - 4(\Delta x)^2}{x^6+3x^5\Delta x+3x^4(\Delta x)^2+x^3(\Delta x)^3} = \frac{-12x^2 - 12x(0) - 4(0)^2}{x^6+3x^5(0)+3x^4(0)^2+x^3(0)^3}$$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{-12x^2}{x^6} = -12x^{2-6}$$

$$= -12x^{-4} \text{ OR } \frac{-12}{x^4}$$

2a $\frac{dx}{(x^2+36)}$

Solution

$$\int \frac{1}{(x^2+36)}\,dx = \int \frac{1}{36\left(\frac{x^2}{36}+1\right)}\,dx = \frac{1}{36}\int \frac{1}{\left(\frac{x^2}{36}+1\right)}\,dx = \frac{1}{36}\int \frac{1}{\left(\frac{x}{6}\right)^2+1}\,dx$$

$u = \frac{x}{6}$

$\frac{du}{dx} = \frac{1}{6}$

$dx = 6\,du$

$$= \frac{1}{36}\int \frac{1}{u^2+1}\,dx = \frac{1}{36}\int \frac{1}{u^2+1}\cdot 6\,du = \frac{\cancel{6}}{\cancel{36}_{6}}\int \frac{1}{u^2+1}\,du$$

Recall, $\int \frac{1}{u^2+1}\,du$ is a standard integral, so it equates to $\arctan u$

$= \frac{1}{6}\arctan u + C \quad ; \quad u = \frac{x}{6}$

$= \frac{1}{6}\arctan(x/6) + C$

$= \frac{\arctan(x/6)+C}{6}$

2b. $\frac{dx}{(x^2+13)}$

Solution

$$\int \frac{1}{(x^2+13)}\,dx = \int \frac{1}{13\left(\frac{x^2}{13}+1\right)}\,dx = \frac{1}{13}\int \frac{1}{\frac{x^2}{13}+1}\,dx = \frac{1}{13}\int \frac{1}{\left(\frac{x}{\sqrt{13}}\right)^2+1}\,dx$$

$u = \frac{x}{\sqrt{13}}$

(Rationalize)

$u = \frac{x}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}x}{13}$

$u = \frac{\sqrt{13}x}{13}$

$\frac{du}{dx} = \frac{\sqrt{13}}{13}$

$dx = \frac{13}{\sqrt{13}} du$

$= \frac{1}{13}\int \frac{1}{u^2+1} dx = \frac{1}{13}\int \frac{1}{u^2+1} \cdot \frac{13}{\sqrt{13}} du = \frac{13}{13\sqrt{13}}\int \frac{1}{u^2+1} du$

Recall, $\int \frac{1}{u^2+1} du$ is a standard integral, so it equates to arctan u

$= \frac{13}{13\sqrt{13}} \arctan u + c$, $u = \frac{\sqrt{13}x}{13}$

$= \frac{\sqrt{13}}{13} \arctan\left(\sqrt{13}x/13\right) + c$

$= \frac{\sqrt{13} \times \arctan\left(\sqrt{13}x/13\right)}{13} + c$

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