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19/scit7/007
Biotechnology

- ① Find the derivative of the following using first principle
 a) $y = \sin(3/x^2)$ b) $y = 4/x^3$
 ② Find the integral of the following
 a) $dx/(x^2+13)$
 b) $dx/(x^2+13)$

Solution

1 a. $y = \sin(3/x^2)$

$y = \sin 3/x^2$

~~$y = 3/x^2$~~ $y = \sin 3x^{-2}$

let $u = 3x^{-2}$

$y = \sin u$

$\frac{dy}{du} \times \Delta u = \cos u \Delta y + y = \sin(u + \Delta u)$

$\Delta y = \sin(u + \Delta u) - y$

$\Delta y = \sin(u + \Delta u) - \sin u$

recall $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$

$A = u + \Delta u$ $B = u$

$\frac{A+B}{2} = \frac{u + \Delta u + u}{2} = \frac{2u + \Delta u}{2} = u + \frac{\Delta u}{2}$

$\frac{A-B}{2} = \frac{u + \Delta u - u}{2} = \frac{\Delta u}{2}$

Hence $\frac{\Delta y}{\Delta u} = \frac{2 \cos(u + \frac{\Delta u}{2}) \sin(\frac{\Delta u}{2})}{\Delta u}$

$\frac{\Delta y}{\Delta u} = \frac{2 \cos(u + \frac{\Delta u}{2}) \sin(\frac{\Delta u}{2}) \times \frac{1}{2}}{\Delta u \times \frac{1}{2}}$

$\frac{\Delta y}{\Delta u} \approx \cos(u + 0)$ Since $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$

$\frac{\Delta y}{\Delta u} = \cos(u + 0) \lim_{\Delta u \rightarrow 0} \frac{(\frac{\Delta u}{2})/\frac{\Delta u}{2}}{\frac{\Delta u}{2}}$

$\frac{dy}{du} = \cos u \times 1$

$\frac{dy}{du} = \cos u$

$$\text{Also } u = \frac{3}{x^2}$$

$$\Delta u + u = \frac{3}{(x+\Delta x)^2}$$

$$\Delta u = \frac{3}{(x+\Delta x)^2} - u$$

$$\Delta u = \frac{3}{(x+\Delta x)^2} - \frac{3}{x^2}$$

$$\Delta u = \frac{3x^2 - 3(x+\Delta x)^2}{x^2(x+\Delta x)^2}$$

$$\Delta u = \frac{3x^2 - 3(x^2 + 2x\Delta x + (\Delta x)^2)}{x^2(x+\Delta x)^2}$$

$$\Delta u = \frac{3x^2 - 3x^2 - 6x\Delta x - 3(\Delta x)^2}{x^2(x+\Delta x)^2}$$

$$\Delta u = \frac{-6x\Delta x - 3(\Delta x)^2}{x^2(x+\Delta x)^2}$$

$$\frac{\Delta u}{\Delta x} = \frac{-6x\Delta x}{x^2(x+\Delta x)^2} \times \frac{1}{\Delta x} - \frac{3(\Delta x)^2}{x^2(x+\Delta x)^2} \times \frac{1}{\Delta x}$$

$$\frac{\Delta u}{\Delta x} = \frac{-6x}{x^2(x+\Delta x)^2} - \frac{3\Delta x}{x^2(x+\Delta x)^2}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta u}{\Delta x} = \frac{-6x}{x^2(x+0)^2} - \frac{3(0)}{x^2(x+0)^2}$$

$$\frac{du}{dx} = \frac{-6x}{x^4} - 0$$

$$\frac{du}{dx} = -6x^{-3}$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$= -6x^{-3} \times \cos u$$

recall $u = \frac{3}{x^2}$

$$\therefore \frac{dy}{dx} = -\frac{6}{x^3} \times \cos\left(\frac{3}{x^2}\right)$$

$$= -\frac{6}{x^3} \cos \frac{3}{x^2}$$

$$16. \quad y = 4/x^3$$

$$y + \Delta y = \frac{4}{(\Delta x + x)^3}$$

$$\Delta y = \frac{4}{(\Delta x + x)^3} - y$$

$$\Delta y = \frac{4}{(\Delta x + x)^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4x^3 - 4(\Delta x + x)^3}{x^3(\Delta x + x)^3}$$

$$\Delta y = \frac{4x^3 - 4[x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3]}{x^3[x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3]}$$

$$\Delta y = \frac{4x^3 - 4x^3 - 12x^2(\Delta x) - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3(\Delta x + x)^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12x^2(\Delta x)}{x^3(\Delta x + x)^3} \times 1 - \frac{12x(\Delta x)^2}{x^3(\Delta x + x)^3} \times 1 - \frac{4(\Delta x)^3}{x^3(\Delta x + x)^3} \times 1$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{-12x^2}{x^3(0+x)^3} - 0 - 0$$

$$\frac{dy}{dx} = \frac{-12x^2}{x^6}$$

$$\frac{dy}{dx} = -12x^{-4}$$

$$(2) \int \frac{1}{x^2+36} dx$$

$$u = x/6 \rightarrow \frac{du}{dx} = 1/6$$

$$dx = 6 du$$

$$= \int \frac{6}{36u^2+36} du$$

$$= \frac{1}{6} \int \frac{1}{u^2+1} du$$

$$\text{also } \int \frac{1}{u^2+1} du = \arctan(u)$$

$$\therefore \frac{1}{6} \int \frac{1}{u^2+1} du = \frac{\arctan(u)}{6}$$

$$\text{recall } u = \frac{x}{6}$$

$$\text{thus } \int \frac{1}{x^2+36} dx = \frac{\arctan\left(\frac{x}{6}\right)}{6} + C$$

$$\text{b) } \int \frac{1}{x^2+13} dx$$

$$u = \frac{x}{\sqrt{13}} \rightarrow \frac{du}{dx} = \frac{1}{\sqrt{13}}$$

$$dx = \sqrt{13} du$$

$$= \int \frac{\sqrt{13}}{13u^2+13} du$$

$$= \frac{1}{\sqrt{13}} \int \frac{1}{u^2+1} du$$

$$\text{also } \int \frac{1}{u^2+1} du = \arctan(u)$$

$$\therefore \frac{1}{\sqrt{13}} \int \frac{1}{u^2+1} du = \frac{\arctan(u)}{\sqrt{13}}$$

$$\text{recall } u = \frac{x}{\sqrt{13}}$$

$$\text{thus } \int \frac{1}{x^2+13} dx = \frac{\arctan\left(\frac{x}{\sqrt{13}}\right)}{\sqrt{13}} + C$$