Name: Opoola Daniel Oluwaseyi
Matric No.: 19/ENG05/056
Department: Mechatronics
Date submitted: 17th April, 2020

1a) $y = \sin 3x^{-2} = \sin \cancel{3u}$

let $u = 3x^{-2}$

$y = \sin \cancel{3u}$

$\frac{du}{dx} = -6x^{-3}$

$f(u) = \sin \cancel{3u}$

$f'u = \lim_{h \to 0} \frac{f(u+h) - f(u)}{h}$ $\qquad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$f'(u) = \lim_{h \to 0} \frac{\sin(u+h) - \sin u}{h}$

$= \lim_{h \to 0} \frac{\sin u \cos h + \sin h \cos u - \sin u}{h}$

$= \lim_{h \to 0} \frac{\sin u [\cos h - 1] + \sin h \cos u}{h}$

$= \sin u \lim_{h \to 0} \frac{[\cos h - 1]}{h} + \cos u \lim_{h \to 0} \frac{\sin h}{h}$

$\lim_{h \to 0} \frac{\sin h}{h} = 1$; $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$

$f'u = \cancel{0} \cancel{+} \cos u$

~~Using chain rule $f'(x) = f'(u) * f'$~~ $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

but

$f'(x) = f'(u) \times du/dx = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \cos u \times (-6x^{-3}) = -6x^{-3} \cos 3x^{-2}$

$= \frac{-6}{x^3} \cos 3/x^2$

$-6(3/x^2)$

y^3

1b) $y = 4/x^3$

If $y = f(x) = \dfrac{4}{x^3}$

note $h = \Delta x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{4}{(x+h)^3} - \frac{4}{x^3}}{h}$$

$$= 4 \lim_{h \to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$= 4 \lim_{h \to 0} \frac{(x+h)^3 - x^3}{((x+h)^3 x^3)\, h}$$

expand $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$$= 4 \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{x^3 . (x+h)^3 . h}$$

$$= 4 \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{x^3 (x+h)^3 \cancel{h}}$$

$$= 4 \lim_{h \to 0} \frac{3x^2 + 3xh + h^2}{x^6 + x^3h^3}$$

$$= 4 \lim_{h \to 0} \frac{3x^2 + 3x(0) + (0)^2}{x^6 + x^3(0)^3}$$

$$= 4 \cdot \frac{3x^2}{x^6} = \frac{12\cancel{x^2}}{\cancel{x^6}} = \frac{12}{x^4}$$

2.) Find the integral of the following

(a) $dx/(x^2+36)$

(b) $dx/(x^2+13)$

(a)
$$\int \frac{dx}{(x^2+36)} = \int \frac{dx}{x^2+6^2}$$

$$x = 6\tan\theta$$

$$\frac{dx}{d\theta} = 6\sec^2\theta$$

$$dx = 6\sec^2\theta \, d\theta$$

$$x^2+6^2 = 6^2\tan^2\theta + 6^2 = 6^2(\tan^2\theta+1)$$
$$= 36\sec^2\theta$$

$$\Rightarrow \int \frac{\cancel{6\sec^2\theta}\, d\theta}{\cancel{36\sec^2\theta}} = \int \frac{d\theta}{6} = \frac{1}{6}\int d\theta$$

$$= \frac{1}{6}[\theta] + c$$

$$= \frac{1}{6}\tan^{-1}\frac{x}{6} + c$$

where $\theta = \tan^{-1} x/a$

and $a = 6$

2b) $\int \frac{dx}{x^2+13} = \int \frac{dx}{x^2+(\sqrt{13})^2}$

$x = \sqrt{13}\tan\theta$

$\frac{dx}{d\theta} = \sqrt{13}\sec^2\theta$

$dx = \sqrt{13}\sec^2\theta\, d\theta$

$x^2+(\sqrt{13})^2 = (\sqrt{13})^2\tan^2\theta + (\sqrt{13})^2 = (\sqrt{13})^2(\tan^2\theta+1)$

$= (\sqrt{13})^2\sec^2\theta$

$= 13\sec^2\theta$

$= \int \frac{\cancel{\sqrt{13}}\,\cancel{\sec^2\theta}\, d\theta}{\cancel{13\sec^2\theta}} = \int \frac{d\theta}{\sqrt{13}} = \frac{1}{\sqrt{13}}\int d\theta$

$= \frac{1}{\sqrt{13}}[\theta] + c$ when $\theta = \tan^{-1} x/\sqrt{13}$

$= \frac{1}{\sqrt{13}} \cdot \tan^{-1}\frac{x}{13} + c$