2b $\int \frac{1}{x^2+13}\,dx$

$u = \frac{x}{\sqrt{13}}$, $du/dx = \frac{1}{\sqrt{13}}$

$dx = \sqrt{13}\,du$

$= \int \frac{\sqrt{13}}{13u^2+13}\,du$

$= \frac{1}{\sqrt{13}} \int \frac{1}{u^2+1}\,du$

also $\int \frac{1}{u^2+1}\,du = \arctan(u)$

$\therefore \frac{1}{\sqrt{13}} \int \frac{1}{u^2+1}\,du = \frac{\arctan(u)}{\sqrt{13}}$

recall $u = \frac{x}{\sqrt{13}}$

$\therefore \int \frac{1}{x^2+13}\,dx = \frac{\arctan\left(\frac{x}{13}\right)}{\sqrt{13}} + C$

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$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-12x^2}{x^3(0+x)^3} - 0 - 0$$

$$\frac{dy}{dx} = \frac{-12x^2}{x^6}$$

$$\frac{dy}{dx} = -12x^{-4}$$

2a $\int \frac{1}{x^2+36}\, dx$

$u = x/6 \quad \& \quad \frac{du}{dx} = \frac{1}{6}$

$dx = 6du$

$$= \int \frac{6}{36u^2+36}\, du$$

$$= \frac{1}{6}\int \frac{1}{u^2+1}\, du$$

also $\int \frac{1}{u^2+1}\, du = \arctan(u)$

$$\therefore \frac{1}{6}\int \frac{1}{u^2+1}\, du = \frac{\arctan(u)}{6}$$

recall $= u = x/6$

$$\int \frac{1}{x^2+36}\, dx = \frac{\arctan(\frac{x}{6})}{6} + C$$

$= -6x^{-3} \times \cos u$

recall $u = 3/x^2$

$\therefore \frac{dy}{dx} = \frac{-6}{x^3} \times \cos(3/x^2)$

$= -\frac{6}{x^3} \cos 3/x^2$

1b $y = 4/x^3$

$y + Dy = \frac{4}{(Dx + x)^3}$

$Dy = \frac{4}{(Dx + x)^3} - y$

$Dy = \frac{4}{(Dx + x)^3} - \frac{4}{\cancel{3}x^3}$

$Dy = \frac{4x^3 - 4(Dx + x)^3}{x^3 (Dx + x)^3}$

$Dy = \frac{4x^3 - 4[x^3 + 3x^2 (Dx) + 3x(Dx)^2 + (Dx)^3]}{x^3 [x^3 + 3x^2 Dx + 3x(Dx)^2 + (Dx)^3}$

$Dy = \frac{\cancel{4x^3} - \cancel{4x^3} - 12x^2 Dx - 12x(Dx)^2 - 4(Dx)^3}{x^3 (Dx + x)^3}$

$\frac{Dy}{Dx} = \frac{-12^2 (Dx)}{x^3 (Dx + x)^3} \times \frac{1}{Dx} - \frac{12x^2 (Dx)^2}{x^3 (Dx + x)^2} \times \frac{1}{Dx} - \frac{4(Dx)^3}{x^3 (Dx + x)^2} \times \frac{1}{Dx}$

$\frac{Dy}{Dx} = \frac{12^2}{x^3 (Dx \times x)^3} - \frac{12x^2 (Dx)}{x^3 (Dx + x)^3} - \frac{4x (Dx)^2}{x^3 (Dx + x)^3}$

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Also $u = \frac{3}{x^2}$

$$Du + u = \frac{3}{(x+Dx)^2}$$

$$Du = \frac{3}{(x+Dx)^2} - u$$

$$Du = \frac{3}{(x+Dx)^2} - \frac{3}{x^2}$$

$$Du = \frac{3x^2 - 3(x+Dx)^2}{x^2(x+Dx)^2}$$

$$Du = \frac{3x^2 - 3(x^2 + 2xDx + (Dx)^2)}{x^2(x+Dx)^2}$$

$$Du = \frac{-6xDx - 3(Dx)^2}{x^2(x+Dx)^2}$$

$$\frac{Du}{Dx} = \frac{-6xDx}{x^2(x+Dx)^2} \times \frac{1}{Dx} - \frac{3(Dx)^2}{x^2(x+Dx)^2} \times \frac{1}{Dx}$$

$$\frac{Du}{Dx} = \frac{-6x}{x^2(x+Dx)^2} - \frac{3Dx}{x^2(x+Dx)^2}$$

$$\lim_{Dx \to 0} \frac{Du}{Dx} = \frac{-6x}{x^2(x+0)^2} - \frac{3(0)}{x^2(x+0)^2}$$

$$du/dx = \frac{-6x}{x^4} - 0$$

$$du/dx = -6x^{-3}$$

$$dy/dx = \frac{du}{dx} \times \frac{dy}{du}$$

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Computer Science
MAT 104

1a $Y = \sin(3/x^2)$

$y = \sin 3x^{-2}$

let $u = 3x^{-2}$, $y = \sin u$

$Dy + y = \sin(u + Du)$

$Dy = \sin(u + Du) - y$

recall $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$A = u + Du \quad B = u$

$\frac{A+B}{2} = \frac{u + Du + u}{2} = \frac{2u + Du}{2} = \frac{Du}{2}$

$\frac{A-B}{2} = \frac{u + Du - u}{2} = \frac{Du}{2}$

Hence $Dy/Du = \frac{2\cos(u + \frac{Du}{2})\sin(\frac{Du}{2}}{Du \times \frac{1}{2}}$

Since $\lim_{\to 0} = 1$

$Dy/Du = \cos(u + 0) \lim_{Du \to 0} (\frac{Du}{2}) / \frac{Du}{2}$

$dy/du = \cos u \times 1$

$= \cos u$

dy/dx