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MAT 104 Assignment

② a. $\int dx/(x^2+36)$

$$\int \frac{dx}{x^2+36} = \int \frac{dx}{x^2+6^2}$$

$x = 6\tan\theta$

$$\frac{dx}{d\theta} = 6\sec^2\theta$$

$dx = 6\sec^2\theta\, d\theta$

$x^2+6^2 = 6^2\tan^2\theta + 6^2 = 6^2(\tan^2\theta+1)$

$= 36\sec^2\theta$

$$\int \frac{6\sec^2\theta\, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{1}{6}\int d\theta$$

$$= \frac{1}{6}[\theta] + c$$

$$= \frac{1}{6}\tan^{-1} x/6 + c$$

b. $$\int \frac{dx}{x^2+13} = \int \frac{dx}{(x^2+(\sqrt{13})^2}$$

$x = \sqrt{13}\tan\theta$

$$\frac{dx}{d\theta} = \sqrt{13}\sec^2\theta$$

$dx = \sqrt{13}\sec^2\theta\, d\theta$

$x^2+(\sqrt{13})^2 = (\sqrt{13})^2\tan^2\theta + (\sqrt{13})^2 = (\sqrt{13})^2(\tan^2\theta+1)$

$= 13\sec^2\theta$

$$\int \frac{\sqrt{13}\sec^2\theta\, d\theta}{13.\sec^2\theta} = \int \frac{\sqrt{13}\, d\theta}{13} \quad \frac{\sqrt{13}}{13}\int d\theta$$

$$= \frac{\sqrt{13}}{13}[\theta] + c$$

$$= \frac{\sqrt{13}}{13}\tan^{-1} x/13 + C$$

(1a) $Y = \sin(3/x^2) \Rightarrow Y = \sin 3x^{-2}$

let $u = 3x^{-2}$

$\Delta y + y = \sin(\Delta u + u)$

$\Delta y = \sin(\Delta u + u) - y$

$\Delta y = \sin(\Delta u + u) - \sin u$

recall $\sin A - \sin B = 2\cos\frac{A+B}{2} \cdot \sin\frac{A-B}{2}$

~~A = Δu+u~~ $A = \Delta u + u,\ B = u$

$\frac{A+B}{2} = \frac{\Delta u + u + u}{2} = \frac{\Delta u + 2u}{2} = u + \frac{\Delta u}{2}$

$\frac{A-B}{2} = \frac{\Delta u + u - u}{2} = \frac{\Delta u}{2}$

Hence $\frac{\Delta y}{\Delta u} = \frac{\cancel{2}\cos\left(u + \frac{\Delta u}{2}\right)\sin\left(\frac{\Delta u}{2}\right) \times 1/\cancel{2}}{\Delta u \times 1/2}$

$\lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} = \cos(u+0) \quad \lim_{\Delta u \to 0} = \frac{\Delta u}{2} \Big/ \frac{\Delta u}{2}$

$\frac{dy}{dx} = \cos u \times 1 = \cos u$

Recall $u = 3x^{-2}$

$u = \frac{3}{x^2}$

$\Delta u + u = \frac{3}{(\Delta x + x)^2}$

$\Delta u = \frac{3}{(\Delta x + x)^2} - u$

$\Delta u = \frac{3}{(\Delta x + x)^2} - \frac{3}{x^2}$

$\Delta u = \frac{3x^2 - 3(\Delta x + x)^2}{x^2(\Delta x + x)^2}$

$\Delta u = \frac{3x^2 - 3(x^2 + 2x\Delta x + \Delta x^2)}{x^2(\Delta x + x)^2} = \frac{3x^2 - 3x^2 - 6x\Delta x - 3\Delta x^2}{x^2(x + \Delta x^2)}$

$\frac{\Delta u}{\Delta x} = \frac{-6x\Delta x\ 3\Delta x^2}{x^2(\Delta x + x)^2} \times \frac{1}{\Delta x}$

$$\frac{\Delta y}{\Delta x} = \frac{-6x\cancel{\Delta x}}{x^2(x+\Delta x)^2} \times \frac{1}{\cancel{\Delta x}} - \frac{3(\Delta x)^{\cancel{2}}}{x^2(x+\Delta x)^2} \times \frac{1}{\cancel{\Delta x}}$$

$$\frac{\Delta y}{\Delta x} = \frac{-6x}{x^2(x+\Delta x)^2} - \frac{3\Delta x}{x^2(x+\Delta x)^2}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-6x}{x^2(x+0} - \frac{3(0)}{x^2(x+0)}$$

$$\frac{du}{dx} = \frac{-6x}{x^4} = -6x^{-3}$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$= -6x^{-3} \cos u$$

$$= -6x^{-3} \cos\left(\frac{3}{x^2}\right)$$

$$= -\frac{6}{x^3} \cos \frac{3}{x^2}$$

b) $Y = 4/x^3$

$$y + \Delta y = \frac{4}{(\Delta x + x)^3}$$

$$\Delta y = \frac{4}{(\Delta x + x)^3} - y$$

$$\Delta y = \frac{4}{(\Delta x + x)^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4x^3 - 4(\Delta x + x)^3}{x^3(\Delta x + x)^3}$$

$$\Delta y = \frac{\cancel{4x^3} - \cancel{4x^3} - 12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(\Delta x + x)^3}$$

$$\frac{\Delta y}{\Delta x} = \frac{-12x^2\cancel{\Delta x} \times 1}{x^3(\Delta x + x)^3 \times \cancel{\Delta x}} - \frac{12x^2\Delta x^2}{x^3(\Delta x + x)^3 \times \Delta x} - \frac{4\Delta x^3}{x^3(\Delta x + x)^3 \times \Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \quad \frac{-12x^2}{x^3(0+x)^3} - 0 - 0$$

$$\frac{dy}{dx} = \frac{-12x^2}{x^6}$$

$$\frac{dy}{dx} = -12x^{-4}$$