$\frac{dx}{x^2}$

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a) $\int \frac{dx}{x^2+36} = \int \frac{dx}{x^2+6^2}$

$x = 6\tan\theta$

$\frac{dx}{d\theta} = 6\sec^2\theta$

$dx = 6\sec^2\theta\, d\theta$

$x^2+36 = (6\tan\theta)^2 + 36$

$= 6^2\tan^2\theta + 36$

$= 6^2(\tan^2\theta + 1)$; recall $\tan^2\theta + 1 = \sec^2\theta$

$= 6^2(\sec^2\theta)$

$= 36\sec^2\theta$

$\int \frac{dx}{x^2+6^2} = \int \frac{6\sec^2\theta\, d\theta}{36\sec^2\theta}$

$= \int \frac{1}{6}\, d\theta$

$= 1/6[\theta] + c$

$= 1/6\tan^{-1}\frac{x}{6} + c$

26. $\int dx / x^2 + 13$

$x = \sqrt{13} \tan\theta$

$\frac{dx}{d\theta} = \sqrt{13} \sec^2\theta$

$dx = \sqrt{13} \sec^2\theta \, d\theta$

$x^2 + 13 = (\sqrt{13} \tan\theta)^2 + 13$

$13 \tan^2\theta + 13$

$= 13 (\tan^2\theta + 1)$; recall $\tan^2\theta + 1 =$

$= 13 \sec^2\theta$

$\therefore \int \frac{dx}{x^2+13} \Rightarrow \int \frac{\sqrt{13} \sec^2\theta \, d\theta}{13 \sec^2\theta}$

$= \int \frac{\sqrt{13}}{13} \, d\theta$

$= \frac{\sqrt{13}}{13} [\theta] + c$

$= \frac{\sqrt{13}}{13} \tan^{-1} \frac{x}{\sqrt{13}} + c$

7a) differentiate from 1st principle:

$y = \sin 3/x^2$

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$y + \Delta y = \frac{\sin 3}{(x+\Delta x)^2}$

$\Delta y = \frac{\sin 3}{x^2 + 2x\Delta x + \Delta x^2} - y$

$\Delta y = \frac{\sin 3}{x^2 + 2x\Delta x + \Delta x^2} - \frac{\sin 3}{x^2}$

Comparing to the general form

$\sin A - \sin B = 2\cos\frac{A+B}{2} \cdot \sin\frac{A-B}{2}$

$A \Rightarrow \frac{3}{x^2 + 2x\Delta x + \Delta x^2}$, $B \Rightarrow \frac{3}{x^2}$

$A+B \Rightarrow \frac{3}{x^2 + 2x\Delta x + \Delta x^2} + \frac{3}{x^2}$

$= \frac{3x^2 + 3x^2 + 6x\Delta x + 3\Delta x^2}{x^4 + 2x3\Delta x + x^2\Delta x^2}$

$A-B \Rightarrow \frac{3}{x^2 + 2x\Delta x} - \frac{3}{x^2}$

$= \frac{3x^2 - 3x^2 - 6x\Delta x - 3\Delta x^2}{x^4 + 2x^3\Delta x + x^2\Delta x^2}$

At $\lim \Delta x \to 0$

$= 2\cos\left(\frac{3x^2 + 3x^2 + 6x(0) + 3(0)^2}{x^4 + 2x^3(0) + x^2(0)^2}\right) - \sin\left(\frac{-6x(0)}{x^4 + 2}\right.$

$= \frac{\cos 6x}{x^4} = \frac{\cos 6}{x^3} \Rightarrow dy/dx$

$du/dx \Rightarrow \sin u$

$= \cos u$

$\therefore dy/dx = dy/du \times du/dx$

$= \cos(6/x^3) \cdot \cos u$

$= \cos\left(\frac{6}{x^3} \cdot u\right)$

$= \cos\left(\frac{6}{x^3} \cdot u\right)$ //