

## Assignment

1a.  $y = \sin\left(\frac{3}{x^2}\right)$

Solution

$$y = \sin\left(\frac{3}{x^2}\right) \quad \cdot \quad f(x) = \sin\left(\frac{3}{x^2}\right) = f(x+\Delta x) = \sin\left(\frac{3}{(x+\Delta x)^2}\right) = \sin\left(\frac{3}{x^2+2x\Delta x+(\Delta x)^2}\right)$$

$$f(x+\Delta x) = \sin\left(\frac{3}{x^2+2x\Delta x+(\Delta x)^2}\right)$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\sin\left(\frac{3}{x^2+2x\Delta x+(\Delta x)^2}\right) - \sin\left(\frac{3}{x^2}\right)}{\Delta x}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = 2 \cos\left(\frac{\frac{3}{x^2+2x\Delta x+(\Delta x)^2} + \frac{3}{x^2}}{2}\right) \sin\left(\frac{\frac{3}{x^2+2x\Delta x+(\Delta x)^2} - \frac{3}{x^2}}{2}\right)$$

$$\begin{aligned} \frac{f(x+\Delta x) - f(x)}{\Delta x} &= 2 \cos\left(\frac{6x^2+6x\Delta x+3(\Delta x)^2}{2x^2(x^2+2x\Delta x+(\Delta x)^2)}\right) \sin\left(\frac{-6x\Delta x-3(\Delta x)^2}{2x^2(x^2+2x\Delta x+(\Delta x)^2)}\right) \\ &= -2 \cos\left(\frac{6x^2+6x\Delta x+3(\Delta x)^2}{2x^2(x^2+2x\Delta x+(\Delta x)^2)}\right) \sin\left(\frac{6x\Delta x+3(\Delta x)^2}{2x^2(x^2+2x\Delta x+(\Delta x)^2)}\right) \end{aligned}$$

1b.  $y = \frac{4}{x^3}$

Solution

$$f(x) = \frac{4}{x^3}$$

Using  $\frac{f(x+\Delta x) - f(x)}{\Delta x}$

$$f(x+\Delta x) = \frac{4}{(x+\Delta x)^3}$$

$$= \frac{4}{(x+\Delta x)(x+\Delta x)^2} = \frac{4}{(x+\Delta x)(x^2+2x\Delta x+(\Delta x)^2)} = \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3}$$

$$f(x+\Delta x) - f(x) = \frac{4}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3} - \frac{4}{x^3}$$

$$f(x+\Delta x) - f(x) = \frac{4x^3 - [4(x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3)]}{x^3(x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3)}$$

$$f(x+\Delta x) - f(x) = \frac{4x^3 - (4x^3 + 12x^2\Delta x + 12x(\Delta x)^2 + 4(\Delta x)^3)}{x^3(x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3)}$$

$$f(x+\Delta x) - f(x) = \frac{4x^3 - 4x^3 - 12x^2\Delta x - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{-12x^2\Delta x - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3} \div \Delta x$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{\Delta x(-12x^2 - 12x\Delta x - 4(\Delta x)^2)}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3} \times \frac{1}{\Delta x}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{-12x^2 - 12x\Delta x - 4(\Delta x)^2}{x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3} = \frac{-12x^2 - 12x(0) - 4(0)^2}{x^3+3x^2(0)+3x(0)^2+(0)^3}$$

$\Delta x \rightarrow 0$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{-12x^2}{x^3} = -12x^{-3}$$

$$= -12x^{-4} \text{ OR } -12$$

$\Delta x \rightarrow 0$

$x^4$

$$2a \quad \frac{dx}{(x^2+36)}$$

Solution

$$= \int \frac{1}{(x^2+36)} dx = \int \frac{1}{36\left(\frac{x^2}{36}+1\right)} dx = \frac{1}{36} \int \frac{1}{\left(\frac{x}{6}\right)^2+1} dx$$

$$u = \frac{x}{6}$$

$$\frac{du}{dx} = \frac{1}{6}$$

$$dx = 6 du$$

$$= \frac{1}{36} \int \frac{1}{u^2+1} dx = \frac{1}{36} \int \frac{1}{u^2+1} \cdot 6 du = \frac{1}{6} \int \frac{1}{u^2+1} du$$

Recall,  $\int \frac{1}{u^2+1} du$  is a standard integral, so it equates to  $\arctan u$

$$= \frac{1}{6} \arctan u + C, \quad u = \frac{x}{6}$$

$$= \frac{1}{6} \arctan\left(\frac{x}{6}\right) + C$$

$$= \frac{\arctan\left(\frac{x}{6}\right) + C}{6}$$

$$2b \quad \frac{dx}{(x^2+13)}$$

Solution

$$\int \frac{1}{(x^2+13)} dx = \int \frac{1}{13\left(\frac{x^2}{13}+1\right)} dx = \frac{1}{13} \int \frac{1}{\left(\frac{x}{\sqrt{13}}\right)^2+1} dx$$

$$y = \frac{x}{\sqrt{13}}$$

(Rationalize)

$$u = \frac{x}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}x}{13}$$

$$u = \frac{\sqrt{13}x}{13}$$

$$\frac{du}{dx} = \frac{\sqrt{13}}{13}$$

$$dx = \frac{13}{\sqrt{13}} du$$

$$= \frac{1}{13} \int \frac{1}{u^2+1} dx = \frac{1}{13} \int \frac{1}{u^2+1} \cdot \frac{13}{\sqrt{13}} du = \frac{13}{13\sqrt{13}} \int \frac{1}{u^2+1} du$$

Recall,  $\int \frac{1}{u^2+1} du$  is a standard integral, so it equals to arctan(u)

$$= \frac{13}{13\sqrt{13}} \arctan(u) + C, \quad u = \frac{\sqrt{13}x}{13}$$

$$= \frac{\sqrt{13}}{13} \arctan\left(\frac{\sqrt{13}x}{13}\right) + C$$

$$= \frac{\sqrt{13} \times \arctan\left(\frac{\sqrt{13}x}{13}\right) + C}{13}$$