NAME: OPUTA CLINTON CHUKWUDUBEM . MATRIC NO 19/ENG09/019
Course: MAT204, General Mathematics III, DEPT: Aeronautics
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1. find the derivative of the following using the first principle

(a) $y = \sin\left(\frac{3}{x^2}\right)$

Solution

let $y = \sin 3x^{-2}$ and let Δy and Δx be small increments in x and y respectively. using the first principle

$$y + \Delta y = \sin 3(x + \Delta x)^{-2}$$

$$\Delta y = \sin 3(x + \Delta x)^{-2} - y \text{ (collecting like terms) since } y = \sin 3x^{-2}$$

$$\Delta y = \sin 3(x+\Delta x)^{-2} - \sin 3x^{-2}$$

$$\Delta y = \sin\left(\frac{3}{(x+\Delta x)^2}\right) - \sin 3x^{-2}, \text{ let } C = \frac{3}{(x+\Delta x)^2} \text{ and } D = 3x^{-2}$$

Recall that $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$$\Delta y = \sin C - \sin D$$

substituting,

$$\Delta y = 2\cos\left[\frac{\left(\frac{3}{(x+\Delta x)^2}\right) + 3/x^2}{2}\right] \cdot \sin\left[\frac{\frac{3}{(x+\Delta x)^2} - 3/x^2}{2}\right]$$

$$\Delta y = 2\cos\left[\frac{\frac{3x^2 + 3(x+\Delta x)^2}{x^2(x+\Delta x)^2}}{2}\right] \cdot \sin\left[\frac{\frac{3x^2 - 3/3(x+\Delta x)^2}{x^2(x+\Delta x)^2}}{2}\right]$$

$$\Delta y = 2\cos\left[\frac{\frac{6x^2 + 6\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}}{2}\right] \cdot \sin\left[\frac{\frac{-6\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}}{2}\right]$$

divide through by Δx

$$\frac{\Delta y}{\Delta x} = \frac{2\cos\left[\frac{\frac{6x^2 + 6\Delta x + 3\Delta x^2}{x^2(x+\Delta x)^2}}{2}\right] \cdot \sin\left[\frac{\frac{-6\Delta x - 3\Delta x^2}{x^2(x+\Delta x)^2}}{2}\right]}{\Delta x}$$

multiply the numerator and denominator by 1/2

$$\frac{\Delta y}{\Delta x} = \frac{\frac{1}{\cancel{2}} \times \cancel{2}\cos\left(\frac{\cancel{6}^{3}x^2}{x^2(x+\Delta x)^2} \times \frac{1}{\cancel{2}} + \frac{\cancel{6}^{3}\Delta x}{x^2(x+\Delta x)^2} \times \frac{1}{\cancel{2}} + \frac{3\Delta x^2}{2(x^2(x+\Delta x)^2)}\right) \cdot \sin\left(\frac{-3\Delta x}{x^2(x+\Delta x)^2} - \frac{3\Delta x^2}{2x^2(x+\Delta x)^2}\right)}{\frac{\Delta x}{2}}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \cos\left(\frac{3x^2}{x^2(x+0)^2} + 0 + 0\right) \cdot \lim_{\Delta x \to 0} \sin(0 - 0) \times 0/2 = 0$$

P.T.O

$$\frac{dy}{dx} = \cos\left(\frac{3x^{2}}{x^{3}x^{2}}\right)$$

$$\frac{dy}{dx} = \frac{\cos 3}{x^{3}}$$

b) $y = \frac{4}{x^3}$

Solution

$y = 4/x^3 = y = 4x^{-3}$ let Δy and Δx be small increments in y and x respectively

$\Delta y + y = 4(x + \Delta x)^{-3}$

$\Delta y = 4(x + \Delta x)^{-3} - y$, Since $y = 4x^{-3}$

$\Delta y = 4(x + \Delta x)^{-3} - 4x^{-3}$, $\Delta y = \frac{4}{(x + \Delta x)^3} - 4x^{-3}$

$$\Delta y = \frac{4}{(x(x+\Delta x)\Delta x(x+\Delta x))(x+\Delta x)} - 4x^{-3}$$ by expansion

$$\Delta y = \frac{4}{(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)} - \frac{4}{x^3}$$, simplifying

$$\Delta y = \frac{4x^3 - 4(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)}{x^3(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)}$$

$$\Delta y = \frac{(4x^3 - 12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3 - 4x^3)}{x^3(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)}$$

$$\Delta y = \frac{-12x^2\Delta x - 12x\Delta x^2 - 4\Delta x^3}{x^3(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)}$$

let $t = 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3$

$$\therefore \Delta y = \frac{-12x^2\Delta x}{x^3 t} - \frac{12x\Delta x^2}{x^3 t} - \frac{4\Delta x^3}{x^3 t}$$

$$\Delta y = \frac{-12\Delta x}{x t} - \frac{12\Delta x^2}{x^2 t} - \frac{4\Delta x^3}{x^3 t}$$

multiply both sides by $1/\Delta x$

$$\frac{\Delta y}{\Delta x} = -\frac{12\Delta x}{x t} \times \frac{1}{\Delta x} - \frac{12\Delta x^2}{x^2 t} \times \frac{1}{\Delta x} - \frac{4\Delta x^3}{x^3 t} \times \frac{1}{\Delta x}$$

$\therefore \frac{\Delta y}{\Delta x} = \frac{-12}{x(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)} = \frac{12\Delta x}{x^3(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 + x^3)} = \frac{4\Delta x^3}{x^3(}$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-12}{x(0+0+0+x^3)} = 0 = 0$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-12}{x^4} = -12x^{-4}$

$\therefore \frac{dy}{dx} = -12x^{-4}$ ans

2. find the integral of the following

(a) $\frac{dx}{(x^2+36)}$

Solution

$\int \frac{1}{x^2+6^2}\,dx = \int \frac{dx}{x^2+6^2}$

$x = 6\tan\theta$

$\frac{dx}{d\theta} = 6\sec^2\theta$, $dx = 6\sec^2\theta\, d\theta$

$x^2+6^2 = 6\tan^2\theta + 6^2 = 6^2(\tan^2\theta + 1)$

factoring after substituting

$\int \frac{6\sec^2\theta\, d\theta}{36\sec^2\theta} = \int \frac{d\theta}{6} = \frac{1}{6}\int d\theta = \frac{1}{6}(\theta) + C$

but $\theta = \tan^{-1} x/6$

$\therefore \frac{1}{6}\tan^{-1}\frac{x}{6} + c$

ans

(b) $\frac{dx}{(x^2+13)}$

Solution

$\int \frac{dx}{x^2+\sqrt{13}} = \int \frac{1}{x^2+\sqrt{13}}\,dx$

$x = \sqrt{13}\tan\theta$

$\frac{dx}{d\theta} = \sqrt{13}\sec^2\theta \quad \therefore dx = d\theta\sqrt{13}\sec^2\theta$

$\therefore x^2+\sqrt{13} = \sqrt{13}\tan^2\theta + \sqrt{13} = \sqrt{13}(\tan^2\theta+1)$ by factorization

Substitute!

$$\int \frac{dx}{x^2+13} = \int \frac{d\theta \cancel{\sqrt{13}}\ \cancel{\sec^2\theta}}{\cancel{13}_{\sqrt{13}}\ \cancel{\sec^2\theta}} = \int \frac{d\theta}{\sqrt{13}} = \frac{1}{\sqrt{13}}\int d\theta$$

$$= \frac{1}{\sqrt{13}}(\theta) + C = \frac{1}{\sqrt{13}}(\theta) + C$$

but $\theta = \tan^{-1}\frac{x}{\sqrt{13}}$

$$\therefore = \frac{1}{\sqrt{13}}\tan^{-1}\frac{x}{\sqrt{13}} + C$$

ans