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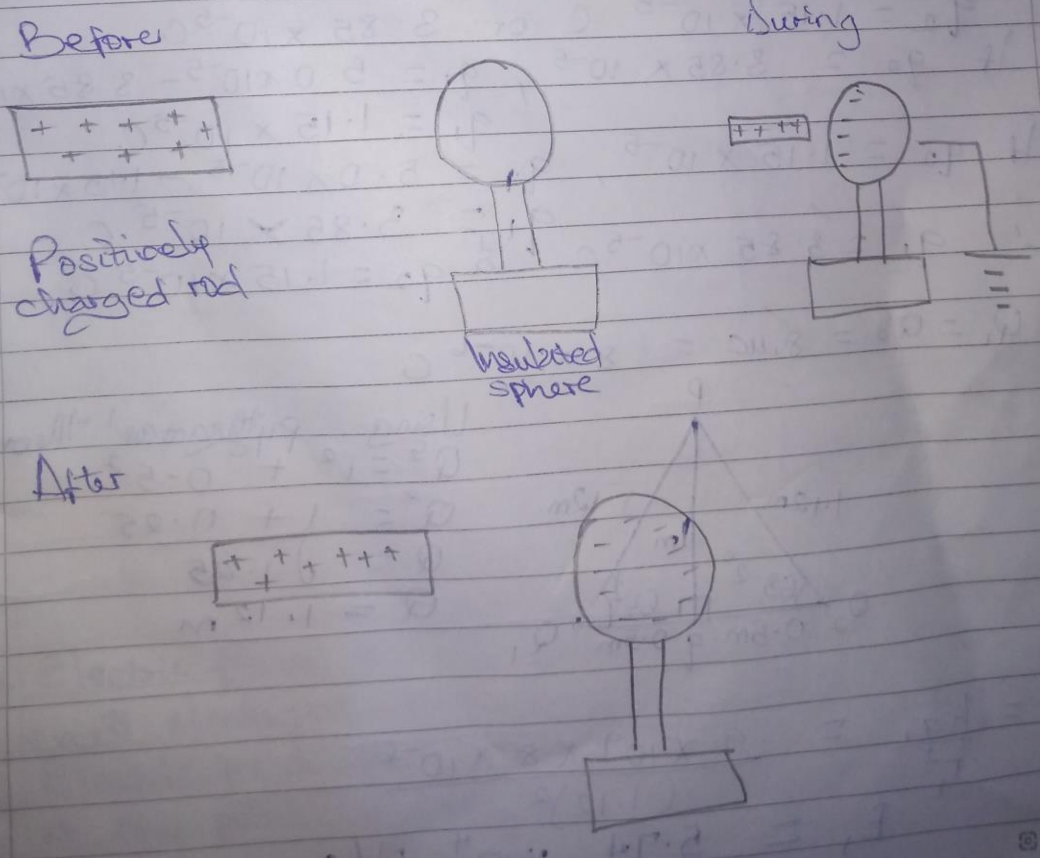
1a. As the charged rod approaches the neutral sphere, the negative charges on the sphere are attracted towards the rod and the positive charges are repelled.
When the sphere is grounded by a wire, ~~electrons~~ electrons move up from the earth and give the sphere a net negative charge.
The grounding wire is removed first and then the charged rod is removed leaving the sphere with a net charge.



b. $q_1 + q_2 = 5.0 \times 10^{-5} C$ ——— ①

$F = 1.0N$, $r = 2.0m$, $k = 9 \times 10^9 Nm^2/C^2$

$F = \frac{kq_1q_2}{r^2}$; $q_1q_2 = \frac{Fr^2}{k} = \frac{1.0 \times (2.0)^2}{9 \times 10^9}$

$q_1q_2 = 4.44 \times 10^{-10} C^2$

from ①, $q_1 = 5.0 \times 10^{-5} C - q_2$

$\therefore (5.0 \times 10^{-5} - q_2) q_2 = 4.44 \times 10^{-10}$

$5.0 \times 10^{-5} q_2 - q_2^2 = 4.44 \times 10^{-10}$

$q_2^2 - 5.0 \times 10^{-5} q_2 + 4.44 \times 10^{-10} = 0$

$q_2 = 1.15 \times 10^{-5} C$ or $3.85 \times 10^{-5} C$

If $q_2 = 3.85 \times 10^{-5}$, $q_1 = 5.0 \times 10^{-5} - 3.85 \times 10^{-5}$

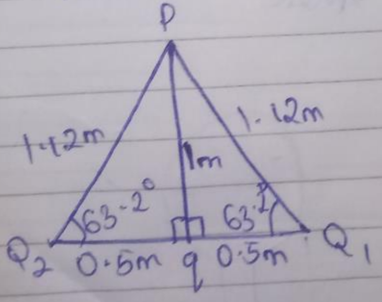
$q_1 = 1.15 \times 10^{-5} C$

If $q_2 = 1.15 \times 10^{-5}$, $q_1 = 5.0 \times 10^{-5} - 1.15 \times 10^{-5}$

$q_1 = 3.85 \times 10^{-5} C$

$\therefore q_1 = 3.85 \times 10^{-5} C$ & $q_2 = 1.15 \times 10^{-5} C$

c. $Q_1 = Q_2 = 8 \mu C = 8 \times 10^{-6} C$



Using pythagoras' theorem

$Q^2 = 1^2 + 0.5^2$

$Q^2 = 1 + 0.25$

$Q = \sqrt{1.25}$

$Q = 1.12 m$

$E_1 = \frac{kq_1}{r_1^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2}$

$E_1 \doteq 5.74 \times 10^4 N/C$

$$E_2 = \frac{kq_2}{r_2^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.12)^2}$$

$$E_2 = 5.74\times10^4 \text{ N/C}$$

$$E_q = \frac{kq}{r_q^2} = \frac{9\times10^9 \times q}{1^2}$$

$$E_q = 9\times10^9 q \text{ N/C}$$

Vector	θ	E_x	E_y
$E_1 = 5.74\times10^4$ N/C	63.2°	$E_1\cos\theta = 2.59\times10^4$ N/C	$E_1\sin\theta = 5.12\times10^4$ N/C
$E_2 = 5.74\times10^4$ N/C	63.2°	$-E_2\cos\theta = -2.59\times10^4$ N/C	$E_2\sin\theta = 5.12\times10^4$ N/C
$E_q = 9\times10^9 q$ N/C	0°	$E_q\cos\theta = 0$ N/C	$E_q\sin\theta = 9\times10^9 q$ N/C

$\Sigma E_x = 0$ N/C , $\Sigma E_y = (102400 + 9\times10^9 q)$ N/C

$$E = \sqrt{(\Sigma E_x)^2 + (\Sigma E_y)^2} = 0$$

$$\sqrt{0^2 + (102400 + 9\times10^9 q)^2} = 0$$

$$0^2 + (102400 + 9\times10^9 q)^2 = 0^2$$

$$102400 + 9\times10^9 q = 0$$

$$\frac{9\times10^9 q}{9\times10^9} = \frac{-102400}{9\times10^9}$$

$$\therefore q = -1.1\times10^{-5}\text{ C} = -11\times10^{-6}\text{ C}$$

$$= -11\,\mu\text{C}$$

2a. Electric field is a region around a charge in which it exerts electrostatic force on other charges while Electric field intensity is the strength of electric field at any point in space.

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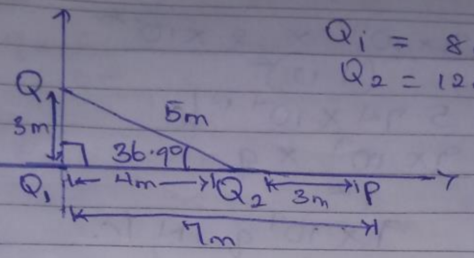
b.



$Q_1 = 8nC = 8 \times 10^{-9} C$

$Q_2 = 12nC = 1.2 \times 10^{-8} C$

i.) $E_1 = \frac{kq_1}{r_1^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2}$

$E_1 = 1.5 N/C$

$E_2 = \frac{kq_2}{r_2^2} = \frac{9 \times 10^9 \times 1.2 \times 10^{-8}}{3^2}$

$E_2 = 12 N/C$

Vector	θ	E_x	E_y
$E_1 = 1.5 N/C$	0°	$E_1 \cos\theta = 1.5\cos 0^\circ = 1.5$	$E_1 \sin\theta = 1.5 \sin 0^\circ = 0$
$E_2 = 12 N/C$	0°	$E_2 \cos\theta = 12 \cos 0^\circ = 12$	$E_2 \sin\theta = 12 \sin 0^\circ = 0$
		$\Sigma E_x = 13.5 N/C$	$\Sigma E_y = 0 N/C$

$E = \sqrt{(\Sigma E_x)^2 + (\Sigma E_y)^2}$

$= \sqrt{(13.5)^2 + 0^2} = \sqrt{(13.5)^2}$

$\therefore E = 13.5 N/C$

ii.) $E_1 = \frac{kq_1}{r_1^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2}$

$E_1 = 8 N/C$

$E_2 = \frac{kq_2}{r_2^2} = \frac{9 \times 10^9 \times 1.2 \times 10^{-8}}{5^2}$

$E_2 = 4.32 N/C$

Vector	θ	E_x	E_y
$E_1 = 8\,N/C$	90°	$E_1\cos\theta = 8\cos 90^\circ = 0$	$E_1\sin\theta = 8\sin 90^\circ = 8$
$E_2 = 4.32\,N/C$	36.9°	$E_2\cos\theta = 4.32\cos 36.9^\circ = 3.5$	$E_2\sin\theta = 4.32\sin 36.9^\circ = 2.6$
		$\Sigma E_x = 3.5\,N/C$	$\Sigma E_y = 10.6\,N/C$

$$E = \sqrt{(\Sigma E_x)^2 + (\Sigma E_y)^2}$$
$$= \sqrt{(3.5)^2 + (10.6)^2}$$
$$E = 11.2\,N/C$$

4a Magnetic flux is the strength of a magnetic field represented by lines of force.

b. $m_e = 9.11 \times 10^{-31}\,kg$, $r = 1.4 \times 10^{-7}\,m$, $B = 3.5 \times 10^{-1}\,weber/m^2$

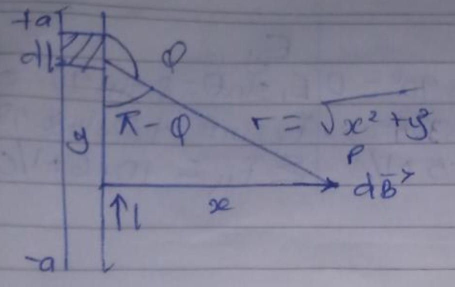
$$\omega = \frac{qB}{m_e} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}},$$
$$\omega = 6.15 \times 10^{10}\,rad/s.$$

5a. Biot-Savart Law is an equation describing the magnetic field generated by a constant electric current

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\vec{l} \times \hat{r}}{r^2}$$ where μ_0 is a constant called permeability of free space.

$$\mu_0 = 4\pi \times 10^{-7}\,T.m/A$$

b.



Applying Biot-Savart law,

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin\theta}{r^2}$$

$$\sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi-\varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi-\varphi)}{x^2+y^2}$$

But $\sin(\pi - \varphi) = \dfrac{x}{(x^2+y^2)^{1/2}}$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}}\, dl$$

$dl = dy$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}}\, dy \quad \text{——} (*)$$

Using special integrals

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

$$\therefore B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2+y^2)^{1/2}} \right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

$(x^2+a^2)^{1/2} \cong a$, as $a \longrightarrow \infty$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$