19/ENG05/007 MAT102

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1 If $A = 5i - 7j - 6k$, $B = j + 4k$; $C = 9i - 4j + k$
find $-8(A+B)\cdot(C-A)$

Solution

$(A+B) = 5i - 6j - 2k$

$-8(A+B) = -40i + 24j + 16k$

$(C-A) = 4i + 3j + 7k$

$\therefore -8(A\cdot B)\cdot(C-A) = \cancel{-160 + (-72)} + -160 + 72 + 112 = \underline{24}$

2 Unit vector tangent to the space curve
$x = -3t$, $y = t^2$, $z = 4t^3$ at the point where $t = 1$

$r = ax\,i + ay\,j + az\,k$

$r = (-3t)i + (t^2)j + (4t^3)k$

$\frac{dr}{dt} = \cancel{-3i + 2tj + 12t^2k}\ -3i + 2tj + 12t^2k$

$\left.\frac{dr}{dt}\right|_{t=1} = -3i + 2(1)j + 12(1)^2k = -3i + 2j + 12k$

$\left|\left.\frac{dr}{dt}\right|_{t=1}\right| = \sqrt{-3^2 + 2^2 + 12^2} = 12.53$

Unit vector tangent $= \dfrac{dr/dt}{\left|\frac{dr}{dt}\right|} = \dfrac{-3i + 2j + 12k}{12.53}$

3 A particle moves along a curve $x = 8t^2$, $y = t^2 - 4t$, $z = t + 1$ where t is time. Find its acceleration

Solution

$r = x_i + y_j + z_k$

$r = (-8t^2)_i + (t^2 - 4t)_j + (t+1)_k$

$r = (-8t^2)_i + (t^2 - 4t)_j + (t+1)_k$

$\frac{dr}{dt} = (-16t)_i + (2t - 4)_j + (1+0)k$

$(2t-4)_j$

Velocity $= -16t_i + (2t-4)_j + (1)k$

$\frac{d^2r}{dt^2} = -16_i + 2_j + 0_k = -16_i + 2_j$

4 $A = i + 2_j - 4k$, $B = 2_i - 3_j + k$, $C = 4_j - 3k$ find

$(A \times B) \times C$

$$(A \times B) = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 2 & -3 & 1 \end{vmatrix} = i\begin{vmatrix} 2 & -4 \\ -3 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & -4 \\ 2 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$= i(-3-4) - j(1-(-8)) + k(-3-4)$

$= -7i - 9_j - 7k$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -7 & -9 & -7 \\ 0 & -4 & -3 \end{vmatrix} = i\begin{vmatrix} -9 & -7 \\ -4 & -3 \end{vmatrix} - j\begin{vmatrix} -7 & -7 \\ 0 & -3 \end{vmatrix} + k\begin{vmatrix} -7 & - \\ 0 & - \end{vmatrix}$$

$= i(27-28) - j(21-0) + k(28-0) = -i - 21_j + 28k$

Given $R = 4\sin 3t\,i + 4e^{3t}\,j + 7t^3 k$ find the integral on R from 0 to 1

Solution

$$\int_0^1 R = \int 4\sin 3t\,i + \int 4e^{3t}\,j + \int 7t^3 k$$

$$= \left(-\frac{4}{3}\cdot \cos 3t\right)i\Big|_0^1 + \left(\frac{4}{3}e^{3t}\right)j\Big|_0^1 + \left(\frac{7t^4}{4}\right)k\Big|_0^1$$

$$= \left(-\frac{4}{3}\cos 3(1)\right)i + \left(\frac{4}{3}e^{3(1)}\right)j + \left(\frac{7}{4}(1)^4\right)k$$

$$= -1.33\,i + 26.8\,j + 1.75\,k$$

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