

3) If $A = 5i - 7j - 6k$, $B = j + 4k$, $C = 9i - 4j + k$, Find $-8(A+B) \cdot (C-A)$.

Soln

$$-8(A+B) = (5i - 7j - 6k) + (j + 4k)$$

$$= (5i - 6j - 2k) \cdot 8$$

$$= (40i + 48j + 16k)$$

$$(C-A) = (9i - 4j + k) - (5i - 7j - 6k)$$

$$= (4i + 3j + 7k)$$

Find $-8(A+B) \cdot (C-A)$.

$$(40i + 48j + 16k) \cdot (4i + 3j + 7k)$$

$$= \begin{vmatrix} 1 & j & k \\ 40 & 48 & 16 \\ 4 & 3 & 7 \end{vmatrix}$$

$$i \begin{vmatrix} 48 & 16 \\ 3 & 7 \end{vmatrix} - j \begin{vmatrix} 40 & 16 \\ 4 & 7 \end{vmatrix} + k \begin{vmatrix} 40 & 48 \\ 4 & 3 \end{vmatrix}$$

$$i(336 - 48) - j(-280 - 64) + k(192 - (-120))$$

$$288i + 344j + 312k$$

4) $x = 3t$, $y = t^2$, $z = 4t^3$

$$r = -3ti + t^2j + 4t^3k$$

$$\frac{dr}{dt} = -3i + 2tj + 12t^2k$$

$$\text{at } t=1 = \frac{dr}{dt} = -3i + 2j + 12k$$

$$\left| \frac{dr}{dt} \right| = \sqrt{(-3)^2 + (2)^2 + (12)^2} = \sqrt{9 + 4 + 144} = \sqrt{157} = \underline{\underline{6.08}}$$

$$\text{Hence } T = \frac{-3i + 2j + 12k}{6.08}$$

5) $x = -8t^2$, $y = t^2 - 4t$, $z = t + 1$

$$r = -8t^2i + (t^2 - 4t)j + (t + 1)k$$

$$\text{velocity vector} = v = \frac{dr}{dt}$$

$$v = -16ti + (2t - 4)j + k$$

$$A = i + 2j - 4k$$

$$B = 2i - 3j + k$$

$$C = 4i - 3k$$

Find $(A+B) \times C$

Soln

$$[A+B] = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 2 & -3 & 1 \end{vmatrix}$$

$$i \begin{vmatrix} 2 & -4 \\ -3 & 1 \end{vmatrix} - j \begin{vmatrix} 1 & -4 \\ 2 & 1 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$$i(2 - 12) - j(-8 - 1) + k(4 - 6)$$

$$-10i + 9j - 2k$$

$$(A+B) \times C = \begin{vmatrix} i & j & k \\ -10 & 9 & -2 \\ 4 & 0 & -3 \end{vmatrix}$$

$$i \begin{vmatrix} 9 & -2 \\ 0 & -3 \end{vmatrix} - j \begin{vmatrix} -10 & -2 \\ 4 & -3 \end{vmatrix} + k \begin{vmatrix} -10 & 9 \\ 4 & 0 \end{vmatrix}$$

$$i(-27 - 6) - j(30 - 8) + k(0 - 40)$$

$$-33i + 22j - 40k$$

$$(A+B) \times C = \sqrt{(-33)^2 + (22)^2 + (-40)^2} = \sqrt{1089 + 484 + 1600} = \sqrt{3173} \approx 56.33$$

$$R = 4\sin 3t i + 4e^{3t} j + 7t^3 k$$

$$|R| dt = \int (4\sin 3t i + 4e^{3t} j + 7t^3 k) dt$$

$$= \int 4\sin 3t dt + \int 4e^{3t} dt + \int 7t^3 dt$$

$$\Rightarrow \int |R| dt = \frac{-4}{3} \cos 3t + \frac{4}{3} e^{3t} + \frac{7}{4} t^4 + C$$