Name: Bando Oluwabo Toluse
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MAT 104

1) $y = \sin\left(\frac{3}{x^2}\right)$

$y = \sin \frac{3}{x^2}$

$y = \sin 3x^{-2}$

let $u = 3x^{-2}$

$y = \sin u$

$\Delta y + y = \sin(u + \Delta u)$

$\Delta y = \sin u + \Delta u) - y$

$= \sin(u + \Delta u) - \sin u$

recall $\sin A - \sin B = 2\cos \frac{A+B}{2} \sin \frac{A-B}{2}$

$A = u + \Delta u \qquad B = u$

$A+B = \frac{u + \Delta u + u}{2} = \frac{2u + \Delta u}{2} = u + \frac{\Delta u}{2}$

$\frac{A-B}{2} = u + \Delta u - u = \frac{\Delta u}{2}$

$= \frac{\Delta y}{\Delta u} = \frac{2\cos\left(u + \frac{\Delta u}{2}\right)}{\Delta u} \sin\left(\frac{\Delta u}{2}\right)$

$\frac{\Delta y}{\Delta u} = \frac{2\cos\left(u + \frac{\Delta u}{2}\right)}{\Delta u \times \frac{1}{2}} \sin\left(\frac{\Delta u}{2}\right) \times \frac{1}{2}$

Since $\lim_{\Delta u \to 0} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

$\frac{\Delta y}{\Delta u} = \cos(u + 0) \quad \lim_{\Delta u \to 0} \left(\frac{\Delta u}{2}\right) / \frac{\Delta u}{2}$

$\frac{\Delta y}{\Delta d} = \cos u \times 1$

$= \cos u$

And $u = \frac{3}{x^2}$

$\Delta u + u = \frac{3}{x + \Delta x^2} - u$

$$\Delta u = \frac{3x^2 - 3(x+\Delta x)^2}{x^2(x+\Delta x)^2}$$

$$\Delta u = \frac{3x^2 - 3(x^2 + 2x\Delta x + (\Delta x)^2)}{x^2(x+\Delta x^2)}$$

$$= \frac{3x^2 - 3(x^2 + 2x\Delta x + (\Delta x)^2)}{x^2(x+\Delta x)^2}$$

Δu ?

$$\Delta u = \frac{-3x^2 + 3x^2 - 6x\Delta x - 3(\Delta x)^2)}{x^2(x+\Delta x)^2}$$

$$\frac{-6x\Delta x - 3(\Delta x)^2}{x^2(x+\Delta x)^2}$$

$$\frac{\Delta u}{\Delta x} = \frac{-6x\Delta x}{x^2(x+\Delta x)^2} \times \frac{1}{\Delta x} - \frac{3(\Delta x)^2}{x^2(x+\Delta x} \times \frac{1}{\Delta x}$$

$$\frac{\Delta u}{\Delta x} = \frac{-6x}{x^2(x+\Delta x)^2} - \frac{3\Delta x}{x^2(x+\Delta x)}$$

$$\lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{-6x}{x^2(x+0)^2} - \frac{3(0)}{x^2(x+0)^2}$$

$$\frac{du}{dx} = \frac{-6x}{x^4} - 0$$

$$\frac{du}{dx} = -6x^{-3}$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$= -6x^{-3} \times \cos u$$

$$u = \frac{3}{x^2}$$

$$-6x^{-3} \times \cos\frac{3}{x^2}$$

$$= -6x^{-3} \cos\frac{3}{x^2}$$

1b) $y = \frac{4}{x^3}$

$$y + \Delta y = \frac{4}{(\Delta x + x)^3}$$

$$\Delta y = \frac{4}{(\Delta x + x)^3} - y$$

$$\Delta y = \frac{4}{(\Delta x + \Delta)^3} - \frac{4}{x^3}$$

$$\frac{4x^3 - 4(\Delta x + x)^3}{x^3(\Delta x + x)^3}$$

$$\Delta y = \frac{4x^3 - 4(x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3]}{x^3(\Delta x + x)^3}$$

$$\frac{-12x^2\Delta x + 12x\Delta x^2 + (\Delta x)^3}{x^3(\Delta x + x)^3}$$

$$\frac{\Delta y}{\Delta x} \quad \frac{12x^2\Delta x}{x^3(\Delta x + x)^3} \times \frac{1}{\Delta x} \quad - \frac{12x^2(\Delta x)^2}{x^3(\Delta x + x)^3} \times \frac{1}{\Delta x} \quad \frac{-4(\Delta x}{x^3(\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-12x^2}{x^3(0 + x)^3} - 0 - 0$$

$$\frac{dy}{dx} = \frac{-12x^2}{x^6}$$

$$\frac{dy}{dx} = -12x^{-4}$$

$$\int \frac{1}{x^2 + 36}\, dx$$

$$u = \frac{x}{6} \rightarrow \frac{du}{dx} = \frac{1}{6}$$

$$dx = 6du$$

$$= \int \frac{6}{36u^2 + 36}\, du$$

$$= \frac{1}{6} \int \frac{1}{u^2 + 1}\, du$$

also $\int \frac{1}{u^2+1} du = \arctan(u)$

$\frac{1}{6} \int \frac{1}{u^2+1} du = \frac{\arctan(u)}{6}$

recall $u = \frac{x}{6}$

thus $\int \frac{1}{x^2+36} dx = \frac{\arctan(\frac{x}{6})}{6} + C$

b) $\int \frac{1}{x^2+13} dx$

$u = \frac{x}{\sqrt{13}} \rightarrow \frac{du}{dx} = \frac{1}{\sqrt{13}}$

$dx = \sqrt{13}\, du$

$= \int \frac{\sqrt{13}}{13u^2+13} du$

$= \frac{1}{\sqrt{13}} \int \frac{1}{u^2+1} du$

also $\int \frac{1}{u^2+1} du = \arctan(u)$

$\frac{1}{\sqrt{13}} \int \frac{1}{u^2+1} du = \frac{\arctan(u)}{\sqrt{13}}$

recall $u = \frac{x}{\sqrt{13}}$

thus $\int \frac{1}{x^2+13} dx = \frac{\arctan\left(\frac{x}{\sqrt{13}}\right)}{\sqrt{13}} + C$