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MECHATRONICS ENGINEERING MAT102

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1 If $A = 5i - 7j - 6k$, $B = j + 4k$, $C = 9i - 4j + k$, find $-8(A+B).(C-A)$

solution

$-8(A+B).(C-A)$

$-8(A+B) = -8[5i - 7j - 6k + j + 4k]$

$-8[5i - 6j - 2k]$

$-8(A+B) = -40i + 48j + 16k$

$(C-A) = 9i - 4j + k - [5i - 7j - 6k]$

$4i + 3j + 7k$

$-8(A+B).(C-A)$

$-40i + 48j + 16k \; . \; 4i + 3j + 7k$

$-160 + 144 + 112$

$\therefore -8(A+B).(C-A) = 96$

2 Find a unit vector tangent to the space curve $x = -3t$ $y = t^2$, $z = 4t^3$ at the point where $t = 1$

solution

$\vec{r} = xi + yj + zk$

$\vec{r} = -3ti + t^2 j + 4t^3 k$

$\frac{d\vec{r}}{dt} = -3i + 2tj + 12t^2 k$

$T = \dfrac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|}$

$\left.\frac{d\vec{r}}{dt}\right|_{t=1} = -3i + 2j + 12k$

$\left|\frac{d\vec{r}}{dt}\right| = \sqrt{-3^2 + 2^2 + 12^2}$

$= \sqrt{9 + 4 + 144}$

$= \sqrt{157}$

12.53

$$T = \frac{-3i + 2tj + 12t^2k}{12.53}$$

3 A particle moves along a Curve $x = 8t^2$, $y = t^2 - 4t$; $z = t+1$, where t is time. find its acceleration

solution

$\vec{r} = xi + yj + zk$

$\vec{r} = 8t^2 i + (t^2 - 4t) j + (t+1) k$

$\frac{d\vec{r}}{dt}$ = velocity = $16ti + (2t - 4) j + k$

$\frac{d^2\vec{r}}{dt^2}$ = acceleration = $16i + 2j$

$\therefore$ acceleration = $16i + 2j$

4 If $A = i + 2j - 4k$ $B = 2i - 3j + k$ $C = 4j - 3k$. find $((A \times B) \times C)$

solution

$(A \times B) \times C$

$$A \times B = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 2 & -3 & 1 \end{vmatrix}$$

$A \times B = (2 - 12) i - (1 - (-8)) j + (-3 - 4) k$

$-10i - 9j - 12k$

$$(A \times B) \times C \quad \begin{vmatrix} i & j & k \\ -10 & -9 & -12 \\ 0 & 4 & -3 \end{vmatrix}$$

$(A \times B) \times C = (27 - (-48)) i - (30 - 0) j + (-40 - 0) k$

$75i - 30j - 40k$

5 Given $R = 4\sin 3t\,i + 4e^{3t}\,j + 7t^3\,k$. find the integral of R with respect to t from 0 to 1

solution

$$R = 4\sin 3t\,i + 4e^{3t}\,j + 7t^3\,k$$

$$\int_0^1 R\,dt = \int_0^1 4\sin 3t\,i\,dt + \int_0^1 4e^{3t}\,j\,dt + \int_0^1 7t^3\,k\,dt$$

$$\cancel{12\cos 3t\,i + 12e^{3t}\,j + 21t\,7}$$

$$-\frac{4}{3}\cos 3t\,i + \frac{4}{3}e^{3t}\,j + \frac{7t^4}{4}k\Bigg|_0^1$$

$$\left[-\frac{4}{3}\cos 3(1)\,i + \frac{4}{3}e^{3(1)}\,j + \frac{7(1)^4}{4}k\right.$$

$$\int_0^1 R\,dt = \left(-\frac{4}{3}\cos 3\right)i + \left(\frac{4}{3}e^3\right)j + \left(\frac{7}{4}\right)k$$

$$\int_0^1 R\,dt = -1.332i + 26.78j + 1.75k$$