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MATRIC NO: 19/SC109/003
DEPT: Industrial Chemistry

100L

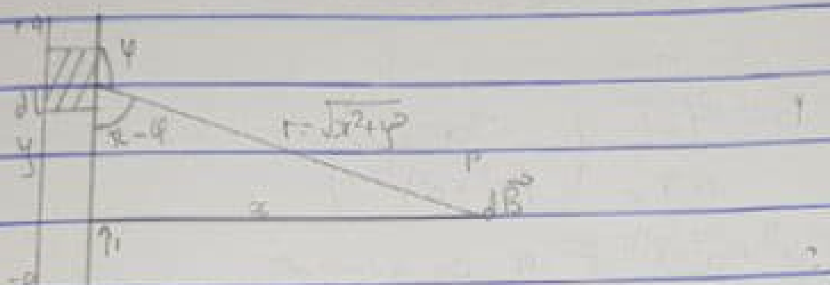
SECTION B

(5a) Bio Savart law is a mathematical expression which illustrates the magnetic field produced by a stable electric current in the particular electro-magnetism of physics. It states that, "the magnetic field is directly proportional to the product permeability of free space (μ_0), the current (I), the change in length ($d\vec{l}$), the radius (r) and inversely proportional to the square of radius (r^2).

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$

$\mu_0 = 4\pi \times 10^{-7}$ T.m/A

(5b)



Applying the Bio-Savart law, we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin\varphi}{r^2} \qquad \sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

From the diagram, $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad \text{--- (1)}$$

But $\sin(\pi - \varphi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}}$ --- (2)

Subst. (2) into (1), we have

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{3/2}}$$

Recall that $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy \quad \text{--- (3)}$$

Using special integrals:

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\frac{y}{(x^2+y^2)^{1/2}}$$

$\because$ Equation (3) becomes

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{2a}{x^2(x^2+a^2)^{1/2}}\right]$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

$$(x^2+a^2)^{1/2} \simeq a, \text{ as } a \to \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

(4a) Magnetic flux is defined as the strength of magnetic field represented by lines of force. It is usually represented by the symbol ϕ.

(4b) $m_e = 9.11\times10^{-31}\,kg$, $q = -1.60\times10^{-19}\,C$

$r = 1.4\times10^{-7}\,m$, $B = 3.5\times10^{-1}$ weber/meter^2

$$\omega = \frac{qB}{m_e}$$

$$\therefore \omega = \frac{-1.60\times10^{-19}\times(3.5\times10^{-1})}{9.11\times10^{-31}}$$

$\therefore \omega =$ ~~$-6\times$~~ -6.147×10^{10} rad/s

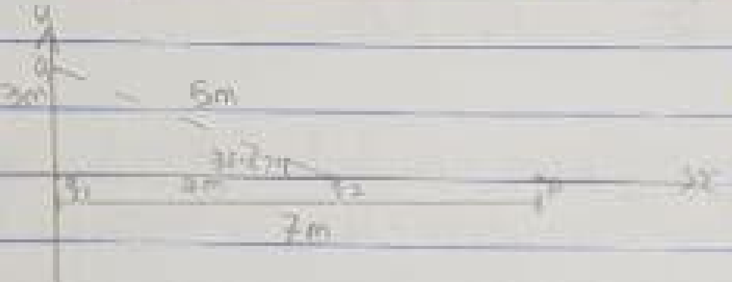
4c We were given the mass of electron $= 9.11\times10^{-31}$ kg, the radius as 1.4×10^{-7} m, the charge as -1.60×10^{-19} C and the magnetic field of 3.5×10^{-1} weber/meter^2 we were asked to find the cyclotron frequency which is also equal to the angular speed. Recall the angular speed, $\omega = \frac{qB}{m_e}$, by substituting each parameter, our angular speed, $\omega = -6.147\times10^{10}$ rad/s. Therefore, the cyclotron frequency $= -6.147\times10^{10}$ ~~rad/s~~ T^{-1}.

SECTION A

2a.

ELECTRIC FIELD INTENSITY	ELECTRIC FIELD
Electric field intensity (E) is the force per unit charge $E = \frac{F(N)}{q_0(C)}$	Electric field is a region of space in which an electric charge will experience an electric force.

b.



$K = 9\times10^9\ Nm^2C^{-2}$, $q_1 = 8\times10^{-9}$ C

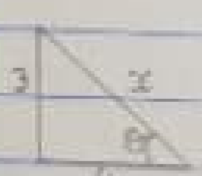
$q_2 = 12\times10^{-9}$ C

i. q_1 to $P = \frac{kq_1}{r^2} = \frac{(9\times10^9)(8\times10^{-9})}{(7)^2} = 1.47$ N/C

q_2 to $P = \frac{kq_2}{r^2} = \frac{(9\times10^9)(12\times10^{-9})}{(3)^2} = 12$ N/C

$\cdot E_{net} = (12 + 1.47) = 13.47 \approx 13.5$ N/C

ii)



Using pythagoras theorem

$x = \sqrt{3^2+4^2} = 5$

$\tan\theta = \frac{3}{4}$; $\tan\theta = 0.75$

$\therefore \theta = \tan^{-1} 0.75 = 36.87^\circ$

q_1 to $Q = \frac{kq_1}{r^2} = \frac{(9\times10^9)(8\times10^{-9})}{(3)^2} = 8$ N/C

q_2 to $Q = \frac{kq_2}{r^2} = \frac{(9\times10^9)(12\times10^{-9})}{(5)^2} = 4.32$ N/C

VECTOR	ANGLE	X - COMPONENT	Y - COMPONENT
q_1 to q = 8 N/C	90°	$8 \cos 90^\circ$ $= 0$ N/C	$8 \sin 90^\circ$ $= 8$ N/C
q_2 to q = 4.32 N/C	36.87°	$4.32 \cos 36.87^\circ = 3.46$ N/C	$4.32 \sin 36.87^\circ$ $= 2.592$ N/C
		$\sum E_x = 3.46$ N/C	$\sum E_y = 10.592$ N/C

$E_{net} = \sqrt{(\sum E_x)^2 + (\sum E_y)^2}$, $E_{net} = \sqrt{(3.46)^2 + (10.592)^2}$

$\therefore E_{net} = 11.14$ N/C

3a) Volume Charge density, $\rho = \frac{dQ}{dV}$ $\quad dQ = \rho\, dV$

ii. Surface Charge density, $\sigma = \frac{dQ}{dA}$ $\quad dQ = \sigma\, dA$

iii. Linear Charge density, $\lambda = \frac{dQ}{dL}$ $\quad dQ = \lambda\, dL$

3b) Electric Potential Difference between 2 points in an electric field is the work done per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in Volt (V) or Joules per Coulomb (J/C).

Work done, $dW = F \cdot dL$ --- (1)

But $F = -q_0 E$ --- (2)

Subst eqn (2) into (1), $dW = -q_0 E dL$ --- (3)

Total work done in moving the test charge from A to B is

$$W(A \to B)_{Ag} = -q_0 \int_A^B E\,dL \quad \text{--- (4)}$$

From the definition of electric potential difference,

$$V_B - V_A = \frac{W(A \to B)_{Ag}}{q_0} \quad \text{--- (5)}$$

Subst eqn (4) into (5)

$$V_B - V_A = -\int_A^B E\,dL \quad \text{--- (6)}$$

(c) 

$q_1 = 10\mu C$, $q_2 = -2\mu C$, $x = 0$, $x = 4$
$V = 0$

For x, $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r^2}\right]$

$0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x} + \frac{q_2}{x+4}\right]$ divide through by $\frac{1}{4\pi\varepsilon_0}$

$0 = \left[\frac{q_1}{x} + \frac{q_2}{x+4}\right]$

$-\frac{q_2}{x+4} = \frac{q_1}{x} = -\left(\frac{-2\times10^{-6}}{x+4}\right) = \frac{10\times10^{-6}}{x}$

$2\times10^{-6}x = 10\times10^{-6}x + 4\times10^{-5}$

$8\times10^{-6}x = -4\times10^{-5}$

$x = \frac{-4\times10^{-5}}{8\times10^{-6}} = -5 \quad \therefore x = -5$

For y, $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{y} + \frac{q_2}{4-y}\right]$ divide through by $\frac{1}{4\pi\varepsilon_0}$

$0 = \left[\frac{q_1}{y} + \frac{q_2}{4-y}\right]$

$-\frac{q_2}{4-y} = \frac{q_1}{y} = -\left[\frac{-2\times10^{-6}}{4-y}\right] = \left[\frac{10\times10^{-6}}{y}\right]$

$2\times10^{-6}y = 4\times10^{-5} - 10\times10^{-6}y$

$12\times10^{-6}y = 4\times10^{-5}$

$y = \frac{4\times10^{-5}}{12\times10^{-6}} = 3.33 \quad \therefore y = 3.33$

For z, $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{4} + \frac{q_2}{4+z}\right]$ divide through by $\frac{1}{4\pi\varepsilon_0}$

$\cancel{0 = \frac{1}{4}}$ $0 = \left[\frac{q_1}{4} + \frac{q_2}{4+z}\right]$

$\frac{-q_2}{4+z} = \frac{q_1}{4} = -\left[\frac{-2\times10^{-6}}{4+z}\right] = \left[\frac{10\times10^{-6}}{4}\right]$

$8\times10^{-6} = 4\times10^{-5} + 10\times10^{-6}z$

$-3.2\times10^{-5} = 10\times10^{-6}z$

$z = \frac{-3.2\times10^{-5}}{10\times10^{-6}} = -3.2$

when $V = 0$, $x = -5$m and 3.33m