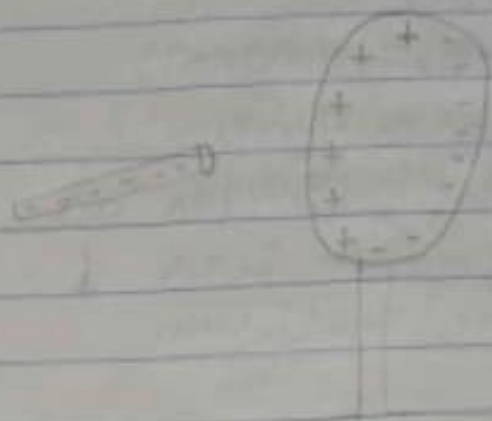


(a) Explain with the aid of diagram how you can produce a negatively charged sphere by method of induction.

Answer



In this process, a charged object is brought near but not touching a neutral conducting object. The presence of a charged object near a neutral conductor will induce (force) electrons within the conductor to move. The movement of electrons leaves an imbalance of charge on opposite sides of the neutral conductor. While the overall object is neutral (i.e. has the same number of electrons & protons), there is an excess of positive charge on one side of the object and an excess of negative charge on the opposite side of the object.

b) We are not given the values of the individual charges. Let them be q_1 and q_2 . The condition on the combined charge of the spheres gives us

$$q_1 + q_2 = 5.0 \times 10^{-5} \text{ C}$$

The next condition concerns the electrostatic force, and so it involves Coulomb's law. Both charges are positive because their signs are positive and they repel each other, thus

$$|q_1| = q_1 \text{ and } |q_2| = q_2$$

Now $F = \frac{kq_1q_2}{r^2} = 1.0 \text{ N}$. We know k and r , so we can

Solve for the value of the product of the charges

$$q_1 q_2 = \frac{(1.6 \times 10^{-19})^2}{K}$$

$$\frac{(0.1)(0.01)}{8.99 \times 10^{11} \text{ N}\cdot\text{m}^2/\text{C}^2} = 4.449 \times 10^{-10} \text{ C}^2$$

Now we have two eqns for the two unknown q_1 and q_2

$$q_2 = 5.0 \times 10^{-5} q_1$$

$$q_1 q_2 = 4.449 \times 10^{-10}$$

$$q_1 (5.0 \times 10^{-5} q_1) = 4.449 \times 10^{-10}$$

$$(5.0 \times 10^{-5} q_1^2) = 4.449 \times 10^{-10}$$

$$q_1^2 - (5.0 \times 10^{-5}) q_1 + 4.449 \times 10^{-10} = 0$$

Use a quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1, \quad b = -5.0 \times 10^{-5} \text{ C}, \quad c = 4.449 \times 10^{-10}$$

$$\frac{5.0 \times 10^{-5} \pm \sqrt{(5.0 \times 10^{-5})^2 - 4(1)(4.449 \times 10^{-10})}}{2}$$

2

$$\frac{5.0 \times 10^{-5} \pm \sqrt{2.5 \times 10^{-9} - 1.7796 \times 10^{-9}}}{2}$$

$$\frac{5.0 \times 10^{-5} \pm \sqrt{2.5 \times 10^{-9} - 1.7796 \times 10^{-9}}}{2}$$

2

$$\frac{5.0 \times 10^{-5} \pm \sqrt{7.204 \times 10^{-10}}}{2}$$

2

$$\frac{5.0 \times 10^{-5} \pm 2.68 \times 10^{-5}}{2}$$

2

$$\therefore x = \frac{5.0 \times 10^{-5} + 2.68 \times 10^{-5}}{2} \quad \text{or} \quad \frac{5.0 \times 10^{-5} - 2.68 \times 10^{-5}}{2}$$

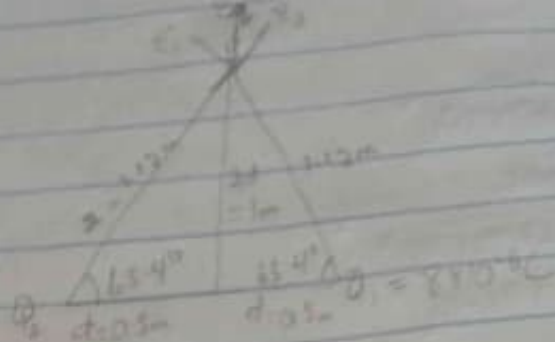
$$x = 3.84 \times 10^{-5} \quad \text{or} \quad 1.16 \times 10^{-5}$$

$$\therefore q_1 = 3.84 \times 10^{-5}, \quad q_2 = 1.16 \times 10^{-5}$$

$$c.) \quad Q_1 = Q_2 = 8 \mu\text{C}$$

$$d = 0.5 \text{ m}$$

determine \vec{Q} if electric field at a point P is zero



Using Pythagoras theorem

$$x^2 = 1^2 + 0.5^2$$

$$x^2 = 1 + 0.25$$

$$x^2 = 1.25$$

$$x = \sqrt{1.25}$$

$$x = 1.12\text{m}$$

Now we find the angle

$$\text{Recall, } \tan \theta = \frac{\text{Opp}}{\text{Adj}}$$

$$\tan \theta = \frac{1}{0.5}$$

$$\tan \theta = 2$$

$$\theta = \tan^{-1}(2)$$

$$\theta = 63.4^\circ$$

$$= \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.12^2} = 57598$$

$$= \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.12^2} = 57598$$

$$= \frac{kq}{r^2} = \frac{9 \times 10^9 \times 9}{1} = 9 \times 10^9$$

vector	angle	x-Comp	y-Comp
57598	63.4°	$-E_1 \times \cos \theta$ = -25700	$E_1 \sin \theta$ = 51323
57598	63.4°	$E_2 \times \cos \theta$ = 25700	$E_2 \sin \theta$ 51323
9×10^9	90°	$E_3 \cos \theta = 0$ $E_3 = 0$	$E_3 \sin \theta = 9 \times 10^9$ $E_y = 102646 + 9 \times 10^9$

$$\text{Magnitude} = \sqrt{(\epsilon_x)^2 + (\epsilon_y)^2}$$

$$\text{Magnitude} = 0$$

$$0 = \sqrt{0^2 + (102646 + 9 \times 10^9 q)^2}$$

Square both sides

$$0^2 = \left(\sqrt{0^2 + (102646 + 9 \times 10^9 q)^2} \right)^2$$

$$0 = 0^2 + (102646 + 9 \times 10^9 q)^2$$

$$\sqrt{0} = \sqrt{0^2 + (102646 + 9 \times 10^9 q)^2}$$

$$0 = 102646 + 9 \times 10^9 q$$

$$0 - 102646 = 9 \times 10^9 q$$

$$-102646 = 9 \times 10^9 q$$

$$q = \frac{-102646}{9 \times 10^9}$$

$$q = -1.14 \times 10^{-5}$$

$$q = -11.4 \times 10^{-6}$$

$$q = \underline{\underline{-11.4 \mu\text{C}}}$$

2a) Electric field is a region around a charge in which it exerts electrostatic force on another charges while the strength of electric field at any point in space is called electric field intensity.

$$2b) E = \frac{kq}{r^2}$$

Due to 8nC

$$E = \frac{8 \times 10^{-9} \times 9 \times 10^9 \times 8 \times 10^{-9}}{3^2}$$

$$E = 8 \times 8 \text{ v/m}$$

Due to 12nC

$$r = 4\text{m} + 3\text{m}$$

$$= 7\text{m}$$

$$E = \frac{kq}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{7^2} = 2.20 \text{ v/m}$$

$$\text{The net electric field at point Q} = 8 + 2.20 \text{ v/m} \\ = 10.2 \text{ v/m}$$

4b) Magnetic flux is defined as the strength of the magnetic field which can be represented as line of forces. It is represented by the symbol ϕ .

$$b) m = 9.11 \times 10^{-31} \text{ kg}$$

$$r = 1.4 \times 10^{-7} \text{ m}$$

$$B = 3.5 \times 10^{-1} \text{ wb/m}^2$$

Rotation frequency = Angular Speed

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$\omega = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.1 \times 10^{-31}} \\ = 6.25 \times 10^{10} \text{ T}^{-1}$$

4c) In the question, there were parameters like

$$m_e = 9.11 \times 10^{-31} \text{ kg}, r = 1.4 \times 10^{-7} \text{ m}, B = 3.5 \times 10^{-1} \text{ wb/m}^2. \text{ We were asked}$$

to find the rotation frequency which is equal to angular speed. Recall that

$$\therefore \frac{10^{-4} \text{ B}}{\text{m}} = \frac{1.6 \times 10^{-7} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-3}}$$
$$= 6.22 \times 10^{-6} \text{ T}$$

5a) Bio-Savart Law states that the magnetic intensity at any point due to a steady current in an infinitely long straight wire is directly proportional to the current and inversely proportional to the distance from point to wire.

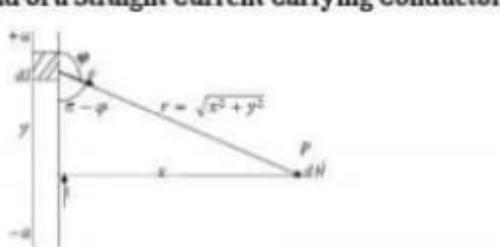


Fig 1: A section of a Straight Current Carrying Conductor

Applying the Biot-Savart law, we find the magnitude of the field dB

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \varphi}{r^2}$$

$$\sin(\pi - \varphi) = \sin \theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad \dots (*)$$

$$\text{But } \sin(\pi - \varphi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \dots (**)$$

Substituting (**) into (*), we have

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 Ix}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \dots (***)$$

Using special integrals:

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

Equation (****) therefore becomes

$$B = \frac{\mu_0 Ix}{4\pi} \left[\frac{y}{x^2(x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 Ix}{4\pi} \left(\frac{2a}{x^2(x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2 + a^2)^{1/2}} \right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P , we consider it infinitely long. That is, when a is much larger than x ,

$$(x^2 + a^2)^{1/2} \cong a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$