Equuatu Duyidem 19/sci01/042
Phy 102 Dept: Computer Science

1a Charging by Induction

Electric charges can be obtained on an object without touching it, by a process called electrostatic induction. Consider a negatively charged rubber rod brought ~~against~~ near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some electrons move to the side of the sphere farthest away from the rod. The region of the sphere nearest the negatively charged rod has an excess of positive charges because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere, as in (fig1.3b), some of the electrons leave the sphere and travel to the earth. If the wire to ground is then removed (fig1.3c), the conducting sphere is left with an excess of induced positive charge. Finally, when the rubber rod is removed from the vicinity of the sphere (fig 1.3d) the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere

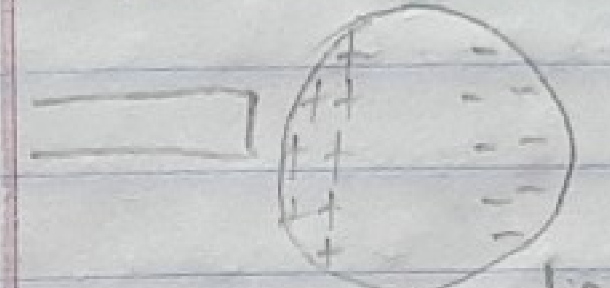


fig 1.3a

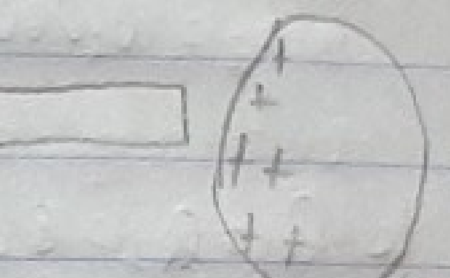


fig1.3c

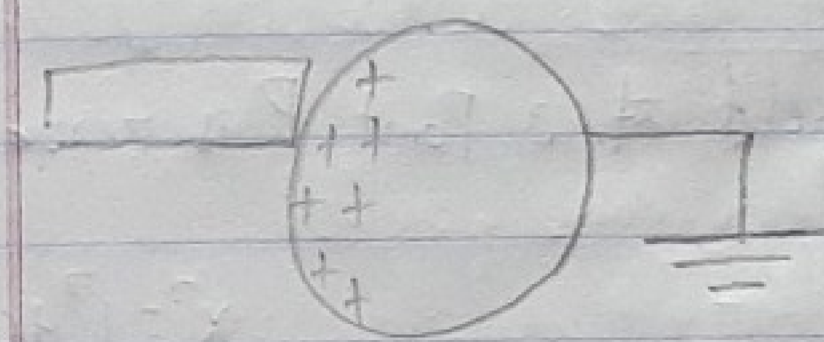


fig 1.3b

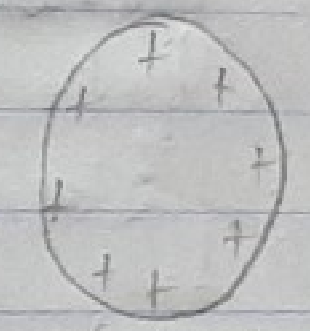


fig 1.3 d

1b $K = 9\times10^{9}$

$q_1 + q_2 = 5\times10^{-5}\,C$

$F = 1N$

$d = 2m$

Charge on each sphere = ?

$F = \frac{kq_1q_2}{r^2}$

$1 = \frac{9\times10^{9}\times(q_1 q_2 5\times10^{-5})}{2^2}$

$4 = 9\times10^{9}\times5\times10^{-5}q_1 + 9_1\times10^{9}q_2$

$4 = 4.5\times10^{5}q_1 + 9\times10^{9}q_2$

Quadratic Equation

$9\times10^9 q_2 - 4.5\times10^5 q_1 + 4 = 0$

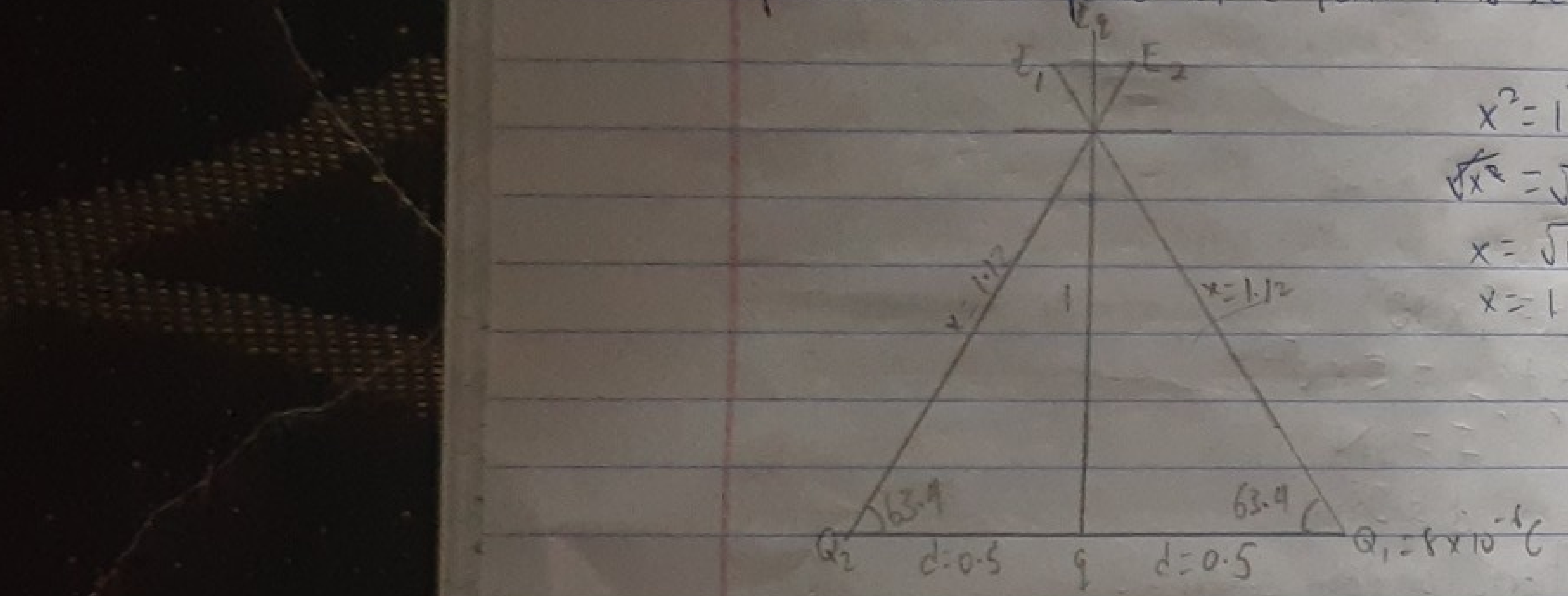
$q_1 = 0.0000111C \simeq 1.11\times10^{-5}C$

$q_2 = 0.000038C \simeq 3.8\times10^{-5}C$

1c $Q_1 = Q_2 = 8\mu C$

$d = 0.5m$

If the electric field at a Point P is zero



$x^2 = 1^2 + 0.5^2$

$\sqrt{x^2} = \sqrt{1.25}$

$x = \sqrt{1.25}$

$x = 1.12$

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{1}{0.5}$

$\theta = \tan^{-1}(2)$

$\theta = 63.4$

$E_1 = \frac{kq_1}{r^2} = \frac{9\times10^9\times8\times10^{-6}}{(1.12)^2} = 57397.9598$

$E_2 = \frac{kq_2}{r^2} = \frac{9\times10^9\times8\times10^{-6}}{(1.12)^2} = 57397.95918$

$E_q = \frac{kq}{r^2} = \frac{9\times10^9\times q}{1} = 9\times10^9 q$

Vector	Angle	X-Component	Y-Component
$E_1 = 57397.95918$	$63.4°$	$E_1\cos\theta = 2570.046785$	$E_1\sin\theta = 5132.262839$
$E_2 = 57397.95918$	$63.4°$	2570.045785	5132.262839
$E_q = 9\times10^9 q$	$90°$	$E_q\cos\theta = 0$	$9\times10^9 q$
		$E_x = 0$	$E_y = 10264.52568$

Magnitude $= \sqrt{(E_x)^2 + (E_y)^2}$

$E_q = \sqrt{(0)^2 + (10264.52568)^2}$

Since $E_q = 0$

$0 = 9\times10^9 q + 10264.52568$

Making q subject of formula

$q = \frac{10264.52568}{9\times10^9}$

$q = 1.140502853\times10^{-16}$

$q = 11.4\mu C$

3a Volume Charge density, $\rho = \frac{dQ}{dv}$; $dQ = \rho dv$

ii Surface Charge density, $\sigma = \frac{dQ}{dA}$; $dQ = \sigma dA$

iii Linear Charge density, $\lambda = \frac{dQ}{dL}$; $dQ = \lambda dL$

3b Electric Potential Difference

The electric potential difference between two points in an electric field can be defined as the workdone per unit charge against electrical forces when a charge is transported from one point to another. It is measured in volts (V) or joules per coulomb (J/C). It is a scalar quantity.

Elemental work done dW is given as

$$dW = f \cdot dL \quad -(i)$$

but $f = -q_0 E \quad -(ii)$

Substituting equation (ii) in (i): $dW = -q_0 E\, dL \quad -(iii)$

Total workdone in moving the test charge from A to B is

$$W(A \to B)_{A_q} = -q_0 \int_A^B E\, dL \quad --(iv)$$

From the definition of electric potential difference, it follows that:

$$V_B - V_A = \frac{W(A \to B)_{A_q}}{q_0} \quad --(v)$$

Putting equation (iv) in (v) yields $V_B - V_A = -\int_A^B E\, dL \quad -(vi)$

Section B

4a Magnetic Flux is defined as the strength of the magnetic field which can be represented by line of forces. It is represented by the symbol ϕ.

4b $m = 9 \times 10^{-31} kg$ $\quad B = 3.5 \times 10^{-1}$ weber/meter^2

$r = 1.4 \times 10^{-7} m$

Cyclotron frequency = angular speed

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$\omega = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$

$$\omega = 6.22 \times 10^{10} T^{-1}$$

4c mass of electron $= 9.11 \times 10^{-31} kg$, radius $= 1.4 \times 10^{-7} m$

magnetic field $= 3.5 \times 10^{-1}$ weber/meter^2

Cyclotron frequency = angular speed

Recall, Angular Speed $\omega = \frac{v}{r} = \frac{qB}{m}$

Substituting $= \omega = \frac{v}{r} = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$

$$= 6.22 \times 10^{10} T^{-1}$$

So cyclotron frequency $= 6.22 \times 10^{10} T^{-1}$, the unit is equal to unit of frequency dimensionally.

5. ~~Biot~~ Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ_0), the current (I), the change in length, the radius are ~~inversely~~ inversely proportional to the square of radius (r^2). It is represented mathematically by:

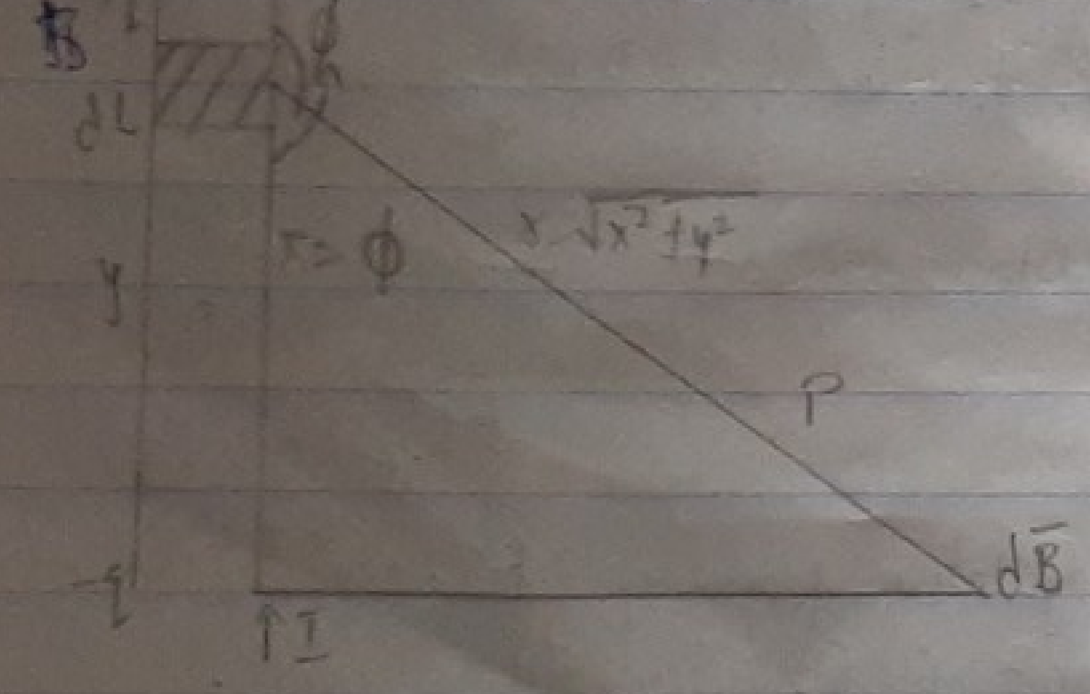
$$dB = \frac{\mu_0 I \, dl \, \hat{r}}{4\pi r^2}$$ where μ_0 is a constant called permeability of free space $\mu = 4\pi \times 10^{-7} \, T.\frac{m}{A}$

Unit of B is weber/meter square

5b. Magnetic Field of a straight current carrying conductor

~~Recall $dl = dy$~~

$$\cancel{B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy}$$



A section of a straight current carrying conductor

Applying the Biot-Savart law, we find the magnitude of the field;

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin\phi}{r^2}$$

$$\sin(\pi - \phi) = \sin\phi$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi-\phi)}{r^2}$$

from diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi-\phi)}{x^2+y^2} \quad \text{---(i)}$$

$$\text{But } \sin(\pi - \phi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}} \quad \text{---(ii)}$$

Substituting (ii) into (i); $B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy \quad \text{---(iii)}$$

Using special integrals, $\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \cdot \frac{y}{(x^2+y^2)^{1/2}}$

Equation (iii) becomes $B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-a}^{a}$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P, we consider it as infinitely long. That B, when a is much larger than x, $(x^2+a^2)^{1/2} \simeq a$, as $a \to \infty$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

- In a physical situation, we have axial symmetry about the y-axis. Thus, at all points in a circle of radius r, around the conductor, the magnitude of B is: $B = \frac{\mu_0 I}{2\pi r}$ --

(magnitude of the magnetic field of flux density B near a lower straight current carrying conductor.)