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ELECT/ELECT

4a

$$\frac{d^2 E_y}{dx^2} = (j\omega\mu\sigma - \omega^2\mu\varepsilon_\cdot) E_y$$

we have → $\frac{d^2 E_y}{dx^2} = \gamma^2 E_y$

where $\gamma = \alpha + j\beta$

$$E_y = E_0 e^{-\gamma x} = E_0 e^{-(\alpha + j\beta)x} \Rightarrow E_0 e^{-\alpha x} e^{-j\beta x}$$

$$\frac{\partial^2 E_y}{\partial x^2} = j\omega\mu\sigma E_y = \gamma^2 E_y \quad \text{or} \quad \gamma^2 = j\omega\mu\sigma$$

$$\gamma = \sqrt{j\omega\mu\sigma} = \alpha + j\beta$$

from $\sqrt{j} = \frac{1+j}{\sqrt{2}}$

$$\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}} \qquad \alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \& \quad \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\therefore E_y = E_0 e^{-\sqrt{\frac{\omega\mu\sigma}{2}}x} e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}x}$$

hence we have ; — $E_y = E_0 e^{-x/\delta} e^{-jx/\delta}$

Hence amplitude of waves decreases as it penetrates a conducting medium exponential by $e^{-x/\delta}$

b) Skin depth's is defined as the depth of penetration of a wave inside a conductor

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \quad \& \; \omega = 2\pi f \quad = \sqrt{\frac{2}{2\pi f\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \quad \text{or} \quad \sqrt{\frac{1}{\pi f\mu\sigma}}$$

c) $f = 10 MHz = 10 \times 10^{6} Hz$

$\sigma = 5.8 \times 10^{7} S/m$

$\mu_0 = 1.257 \times 10^{-6}$

$\mu_r = 1$

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

where

$\mu = \mu_r \times \mu_0 \Rightarrow$

$1 \times 1.25 \times 10^{-6}$

hence

$$\delta = \frac{1}{\sqrt{\pi \times 10 \times 10^{6} \times 1.257 \times 10^{-6} \times 5.8 \times 10^{7}}}$$ $2.09 \times 10^{-5} m$

(7)

$a = 3mm = 0.003m$

$b = 10mm = 0.01 m$

$\varepsilon_0 = 8.85 \times 10^{-12} F/m$ $\mu_0 = 1.257 \times 10^{-6} H/m$

$$C = \frac{2\pi \varepsilon_0}{\log_e b/a} \Rightarrow \frac{2 \cdot \pi \times 8.85 \times 10^{-12}}{\lg_e [0.01/0.003]} = 4.61 \times 10^{-11} F/m$$

b)

Inductance per metre

$$L = \frac{\mu_0}{2\pi} \log_e [b/a] \Rightarrow \frac{1.257 \times 10^{-6}}{2\pi} \times \lg_e [0.01/0.003]$$

$2.40 \times 10^{-7} H/m$

c

Characteristic impedance

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{2.4\times10^{-7}}{4.61\times10^{-11}}}$$

$$Z_0 = 72.15\,\Omega$$

phase velocity

$$V_p = \frac{1}{\sqrt{L\cdot C}}$$

$$V_p = \frac{1}{\sqrt{(2.4\times10^{-7})(4.61\times10^{-11})}}$$

$$V_p = 2.99\times10^{8} \simeq 3.0\times10^{8}\ V$$