

1) If  $A = 5i - 7j - 6k$ ,  $B = j + 4k$ ,  $C = 9i - 4j + k$   
 Find:  $-8(A+B) \cdot (C-A)$

Solution

$$\begin{aligned} -8(A+B) &= -8(5i - 7j - 6k + j + 4k) \\ &= -8(5i - 6j - 2k) \\ &= -40i + 48j + 16k \end{aligned}$$

$$\begin{aligned} (C-A) &= (9i - 4j + k - 5i + 7j + 6k) \\ &= 4i + 3j + 7k \end{aligned}$$

$$-8(A+B) \cdot (C-A)$$

$$= (-40i + 48j + 16k) \cdot (4i + 3j + 7k) = -160i + 144j + 112k$$

$$-8(A+B) \cdot (C-A) = \underline{\underline{-160i + 144j + 112k}}$$

2) Find a unit vector tangent to the space curve

$$x = -3t, \quad y = t^2, \quad z = 4t^3, \quad \text{at } t = 1$$

$$r = -3ti + t^2j + 4t^3k$$

$$\frac{dr}{dt} = -3i + 2tj + 12t^2k$$

at  $t = 1$

$$\frac{dr}{dt} = -3i + 2j + 12k$$

$$\left| \frac{dr}{dt} \right|_{t=1} = \sqrt{(-3)^2 + (2)^2 + (12)^2} = \sqrt{9 + 4 + 144} = \sqrt{157} \approx 12.5$$

$$T = \frac{3i + 2j + 12k}{12.5}$$

3)  $x = -8t^2$ ,  $y = t^2 - 4t$ ,  $z = t + 1$

find  $\frac{d^2A}{dt^2}$  (acceleration)

$$\frac{dA}{dt} = -16t^2 \mathbf{i} + (2t - 4) \mathbf{j} + 2t \mathbf{k}$$

$$\frac{d^2 A}{dt^2} = \underline{\underline{-16 \mathbf{i} + 2 \mathbf{j}}}$$

$$4) A = \mathbf{i} + 2\mathbf{j} - 4\mathbf{k}, B = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}, C = 4\mathbf{j} - 3\mathbf{k}$$

$$(A \times B) \times C$$

$$(A \times B) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -4 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= \mathbf{i} \begin{vmatrix} 2 & -4 \\ -3 & 1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & -4 \\ 2 & 1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$$= \mathbf{i} (2 - 12) - \mathbf{j} (1 + 8) + \mathbf{k} (-3 - 4)$$

$$= \mathbf{i} (-10) - \mathbf{j} (9) + \mathbf{k} (-7)$$

$$(A \times B) = -10\mathbf{i} - 9\mathbf{j} - 7\mathbf{k}$$

$$(A \times B) \times C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -10 & -9 & -7 \\ 0 & 4 & -3 \end{vmatrix}$$

$$= \mathbf{i} \begin{vmatrix} -9 & -7 \\ 4 & -3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} -10 & -7 \\ 0 & -3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} -10 & -9 \\ 0 & 4 \end{vmatrix}$$

$$= \mathbf{i} (27 + 28) - \mathbf{j} (30 - 0) + \mathbf{k} (-40 - 0)$$

$$= \mathbf{i} (55) - \mathbf{j} (30) + \mathbf{k} (-40)$$

$$(A \times B) \times C = \underline{\underline{55\mathbf{i} - 30\mathbf{j} - 40\mathbf{k}}}$$

$$\Rightarrow \int_0^1 R = \int_0^1 4 \sin st \mathbf{i} + 4 e^{st} \mathbf{j} + 7t^3 \mathbf{k} \dots$$

$$\approx \left| -\frac{4}{3} \cos t \mathbf{i} + \frac{4}{3} e^{3t} \mathbf{j} + \frac{7t^4}{4} \mathbf{k} \right| \quad [8 + 131]$$

$$\approx \left| -\frac{4}{3} \cos(1) \mathbf{i} + \frac{4}{3} e^{3(1)} \mathbf{j} + \frac{7(1)^4}{4} \mathbf{k} \right|$$

$$\int_0^1 R = \underline{\underline{-\frac{4}{3} \cos 1 \mathbf{i} + \frac{4}{3} e^3 \mathbf{j} + \frac{7}{4} \mathbf{k}}}$$