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17/Eng 04/052
Electrical/Electronics
Electromag

Question 4a

From the equation

$$\frac{d^2 E_y}{dx^2} = (j\omega \mu \sigma - \omega^2 \mu \varepsilon) E_y = \gamma^2 E_y$$

where γ is the propagation constant
∴ γ has both real & imaginary constants

$$\gamma = \alpha + j\beta$$

$E_y = E_0 e^{-\gamma x} = E_0 e^{-\alpha x} e^{-j\beta x}$ (Suppressing the $e^{j\omega t}$ form) → for a wave travelling in a conducting medium

↓

where conductivity $\sigma >> \omega \varepsilon$

$$\frac{d^2 E_y}{dx^2} = j\omega \mu \sigma E_y = \gamma^2 E_y \quad (11)$$

$$\gamma^2 = j\omega\mu\sigma \quad (11) \qquad \sqrt{j} = \frac{1+j}{\sqrt{2}}$$

$$\gamma = \sqrt{j\omega\mu\sigma} = \alpha + j\beta$$

For the real & imaginary part of γ then we have

$$\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

with $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ and $\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

then Eqn 10 becomes
$E_y = E_0\, e^{-\sqrt{\frac{\omega\mu\sigma}{2}}\, x} \cdot e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}\, x}$

Substituting
$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}} \qquad \omega = \pi f$$

then E_y
$$E_y = E_0\, e^{-x/\delta}\, e^{-jx/\delta}$$

The equation shows the amplitude of the wave dynamic exponentially as it decreases.

(b) Skin depth is a measure of how closely electric current flows along the surface of a material

$$\delta = \sqrt{2/\omega\mu\varepsilon}$$
where $\omega = 2\pi f$

$$= \sqrt{\frac{2}{2\pi f\mu\varepsilon}} = \sqrt{\frac{1}{\pi f\mu\varepsilon}}$$

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \text{ or } \sqrt{\frac{1}{\pi f\mu\varepsilon}}$$

Number 7

$b = 10mm \simeq 0.01m$

$a = 3mm = 0.003m$

$\varepsilon_0 = 8.85 \times 10^{-12} n/m$

$\mu_0 = 1.257 \times 10^{-6} H/m$

(a) Capacitance per metre

$$C = \frac{2\pi\varepsilon_0}{\log_e b/a}$$

$$C = \frac{2\pi \times 8.85 \times 10^{-12}}{\log_e \frac{0.01}{0.003}}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12}}{\log 28.03}$$

$$C = 3.84 \times 10^{-11} f/m$$

(b) Inductance per metre, l

$$L = \frac{\mu_0}{2\pi} \log_e b/a$$

$$L = \frac{1.257 \times 10^{-6}}{2\pi} \log_e 0.01/0.03$$

$1.257 \times 10^{-6} \log 28.03$

$L = 2.90 \times 10^{-7} H/m$

Scanned with CamScanner

(c) Characteristic Impedance

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{2.90 \times 10^{-7}}{3.84 \times 10^{-11}}}$$

$$Z_0 = 86.90 \Omega$$

(d)

phase velocity, V_p

$$V_p = \frac{1}{\sqrt{LC}}$$

$$V_p = \frac{1}{\sqrt{(2.90 \times 10^{-7}) \times (3.84 \times 10^{-11})}}$$

$$V_p = 299664563.4$$

$$\approx 29.10 \times 10^{7}\ \text{m/s}$$