

NAME: OKON, ESTHER STEPHEN

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DEPARTMENT: MBBS

SECTION A

QUESTION 2a

Electric field

An electric field is a region of space in which an electric charge will experience an electric force.

Electric field intensity

This can be defined as the force per unit charge.

2b

$$\vec{E} = \frac{kq_1}{r_1^2} + \frac{kq_2}{r_2^2}$$

$$= \frac{kq_1}{x^2} + \frac{kq_2}{(x-9)^2}$$

$$= \frac{(8.99 \times 10^9)(8 \times 10^{-9})}{(7)^2}$$

$$+ \frac{(8.99 \times 10^9)(12 \times 10^{-9})}{(3)^2}$$

$$= \frac{71.92}{49} + \frac{107.88}{9}$$

$$= 13.45$$

$$\approx 13.5 \text{ N/C}$$

$$\vec{E} = \frac{kq_1}{r_1^2} + \frac{kq_2}{r_2^2}$$

$$= \frac{kq_1}{x^2} + \frac{kq_2}{(a-x)^2}$$

$$= \frac{(8.99 \times 10^9)(8 \times 10^{-9})}{(3)^2}$$

$$+ \frac{(8.99 \times 10^9)(12 \times 10^{-9})}{(1)^2}$$

$$= \frac{71.92}{9} + \frac{107.88}{1}$$

$$= 11.2 \text{ N/C}$$

QUESTION 3a

i) Volume charge density, $\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$

ii) Surface charge density, $\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$

iii) Linear charge density, $\lambda = \frac{dQ}{dL} \rightarrow dQ = \lambda dL$

3b

The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in volt (V) or Joules per coulomb (J/C). Electric potential difference is a scalar quantity. Mathematically,

$$V_B - V_A = \frac{W(A \rightarrow B)}{q_0}$$

3c

i) to the left ($x < 0$)

$$V = \frac{kQ}{r}$$

$$V = \frac{k \times 10 \times 10^{-6}}{x} - \frac{k \times 2 \times 10^{-6}}{4+x}$$

$$0 = \frac{9 \times 10^9 \times 10 \times 10^{-6}}{x} - \frac{9 \times 10^9 \times 2 \times 10^{-6}}{4+x}$$

$$\frac{9 \times 10^9 \times 2 \times 10^{-6}}{4+x} = \frac{9 \times 10^9 \times 10 \times 10^{-6}}{x}$$

$$\frac{2}{4+x} = \frac{10}{x}$$

$$2x = 40 + 10x$$

Collect like terms

$$10x - 2x = -40$$

$$8x = -40$$

Divide both sides by 8

$$x = -5\text{m}$$

ii) to the right

$$0 = \frac{10 \times 10^{-6} \times 9 \times 10^9}{x} - \frac{2 \times 10^{-6} \times 9 \times 10^9}{x-4}$$

$$\frac{2 \times 10^{-6} \times 9 \times 10^9}{x-4} = \frac{10 \times 10^{-6} \times 9 \times 10^9}{x}$$

$$\frac{2}{x-4} = \frac{10}{x}$$

$$2 \cdot x = 10x - 40$$

$$10x - 2x = 40$$

$$8x = 40$$

$$x = 5\text{m}$$

∴ the points are 5m and -5m

SECTION B

QUESTION 4 a

Magnetic flux is defined as the strength of magnetic field represented by lines of force. It is usually represented by the symbol Φ .

4b

$$M = 9 \times 10^{-31} \times 9.11 \times 10^{-31} \text{ kg}$$

$$r = 1.4 \times 10^{-7} \text{ m}$$

$$B = 3.5 \times 10^{-17} \text{ T}$$

$$q = 1.6 \times 10^{-19}$$

$$\omega = ?$$

$$\omega = \frac{qB}{m}$$

$$\omega = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-17}}{9.11 \times 10^{-31}}$$

$$= 6.147 \times 10^{10} \text{ rad/s}$$

QUESTION 5a

Biot-Savart law states that it is a mathematical expression which illustrates the magnetic field produced by a stable electric current in a particular electromagnetism of physics

Substituting (5) into (4), we have

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2+y^2)^{3/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2+y^2)^{3/2}} dy \quad \text{--- (6)}$$

Using special integrals

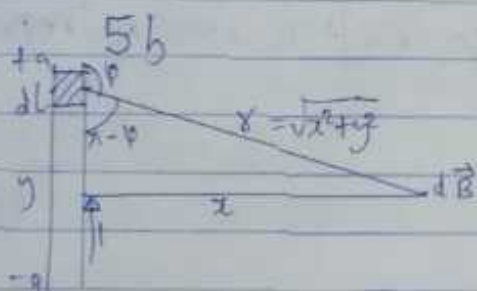
$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

Equation (6) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2+y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$



Applying the Biot-Savart law, we find the magnitude of the field dB

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \phi}{r^2}$$

$$\sin(\pi - \phi) = \sin \phi$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (Pythagorean theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{x^2 + y^2} \quad \text{--- (7)}$$

$$\text{But } \sin(\pi - \phi) = \frac{x}{\sqrt{x^2 + y^2}}$$

$$= \frac{x}{(x^2 + y^2)^{1/2}} \quad \text{--- (8)}$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long. That is when a is much larger than x

$$(x^2 + a^2)^{1/2} \approx a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

\therefore The magnitude of B is

$$B = \frac{\mu_0 I}{2\pi r}$$