NAME: ALEGBELEYE OLUWATOSIN OLUWAPELUMI

DEPARTMENT: COMPUTER SCIENCE

MATRIC NUMBER: 19/SCI01/015

ASSIGNMENT

1 If $A = 5i - 7j - 6K$, $B = j + 4K$, $C = 9i - 4j + K$, find $-8(A+B)\cdot(C-A)$

Solution

$$-8(A+B) = -8[(5i - 7j - 6K) + (j + 4K)]$$

$$= -8[5i - 6j - 2K]$$

$$= [-40i + 48j + 16K]$$

$$(C-A) = [9i - 4j + K] - [5i - 7j - 6K]$$

$$= [9i - 4j + K - 5i + 7j + 6K]$$

$$= [4i + 3j + 7K]$$

$$-8(A+B)\cdot(C-A) = [-40i + 48j + 16K]\cdot[4i + 3j + 7K]$$

$$= -160 + 144 + 112$$

$$= 96$$

2. Find a unit vector tangent to the space curve $x = -3t$, $y = t^2$ and $z = 4t^3$ at the point where $t = 1$.

Solution

$$T = \frac{\frac{dr}{dt}}{\left|\frac{dr}{dt}\right|}$$

$$r = xi + yj + zK$$

$$r = -3ti + t^2j + 4t^3K$$

$$\frac{dr}{dt} = -3i + 2tj + 12t^2K$$

at $t = 1$, $\frac{dr}{dt} = -3i + 2tj + 12t^2K$

$$\frac{dr}{dt} = -3i + 2(1)j + 12(1)^2 K$$

$$\frac{dr}{dt} = -3i + 2j + 12K$$

$$\left|\frac{dr}{dt}\right|_{t=1} = \sqrt{(-3)^2 + (2)^2 + (12)^2}$$

$$= \sqrt{9 + 4 + 144}$$

$$= \sqrt{157} = 12.53$$

Hence, $T = \dfrac{-3i + 2j + 12K}{12.53}$

(3.) A particle moves along a curve, $x = -8t^2$, $y = t^2 - 4t$, $z = t + 1$, where $t$ is time. Find its acceleration.

Solution

$x = -8t^2$, $y = t^2 - 4t$ ; $z = t + 1$

$r = xi + yj + zK$

$r = -8t^2 i + (t^2 - 4t)j + (t+1)K$

$$\frac{dr}{dt} = -16t\, i + (2t - 4)j + K$$

Acceleration $= \cancel{\frac{d^2r}{dt^2}} = \cancel{-16i+2j} \cdot \dfrac{d^2r}{dt^2} = -16i + 2j$

(4.) If $A = i + 2j - 4K$, $B = 2i - 3j + K$, $C = 4j - 3K$. Find $[(A \times B) \times C]$

Solution

$$(A \times B) = \begin{vmatrix} i & j & K \\ 1 & 2 & -4 \\ 2 & -3 & 1 \end{vmatrix}$$

$$(A \times B) = i \begin{vmatrix} 2 & -4 \\ -3 & 1 \end{vmatrix} - j \begin{vmatrix} 1 & -4 \\ 2 & 1 \end{vmatrix} + K \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$(A\times B) = i(2-(12)) - j(1-(-8)) + k(-3-4)$

$= i(2-12) - j(1+8) + k(-7)$

$= -10i - 9j - 7k$

$$[(A\times B)\times C] = \begin{vmatrix} i & j & k \\ -10 & -9 & -7 \\ 0 & 4 & -3 \end{vmatrix}$$

$$= i\begin{vmatrix} -9 & -7 \\ 4 & -3 \end{vmatrix} - j\begin{vmatrix} -10 & -7 \\ 0 & -3 \end{vmatrix} + k\begin{vmatrix} -10 & -9 \\ 0 & 4 \end{vmatrix}$$

$= i(27-(-28)) - j(30-0) + k(-40-0)$

$= i(27+28) - j(30) + k(-40)$

$= 55i - 30j - 40k$

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(5) Given $R = 4\sin 3t\, i + 4e^{3t} j + 7t^3 k$, find the integral of $R$ with respect to $t$ from 0 to 1.

Solution

$R = 4\sin 3t\, i + 4e^{3t} j + 7t^3 k$

$\int_0^1 R = \int 4\sin 3t\, i + \int 4e^{3t} j + \int 7t^5 k$

$\int_0^1 R = 4i\int \sin 3t + 4j\int e^{3t} + 7k\int t^3$

$\int_0^1 R = 4i \times \frac{-\cos(3t)}{3} + 4j \times \frac{e^{3t}}{3} + 7k \times \frac{t^4}{4}$

$\int_0^1 R = \frac{-4\cos(3t) i}{3} + \frac{4e^{3t} j}{3} + \frac{7t^4 k}{4}$

$$\int_0^1 R = \left[\frac{-4\cos(3(1)) i}{3} + \frac{4e^{3(1)} j}{3} + \frac{7(1)^4 k}{4}\right] - \left[\frac{-4\cos(3(0)) i}{3} + \frac{4e^{3(0)} j}{3} + \frac{7(0)^4 k}{4}\right]$$

$$\int_0^1 R = \left[\frac{-4\cos(3) i}{3} + \frac{4e^{3} j}{3} + \frac{7k}{4}\right] + \frac{4\cos(0) i}{3} + \frac{4e^{0} j}{3} + \frac{0k}{4}$$

$$\int_1^0 R = \left[\frac{-4\cos(3)i}{3} + \frac{4\cos(0)i}{3}\right] + \left[\frac{4e^3 j}{3} + \frac{4e^0 j}{3}\right] + \left[\frac{7K}{4} + \frac{0K}{4}\right]$$

$$\int_1^0 R = \frac{-4\cos(3)i + 4\cos(0)i}{3} + \frac{4e^3 j + 4e^0 j}{3} + \frac{7K + 0K}{4}$$

$$\int_1^0 R = \frac{-4\cos(3)i + 4i}{3} + \frac{4e^3 j + 4j}{3} + \frac{(7+0)K}{4}$$

$$\int_1^0 R = \frac{[-4\cos(3) + 4]}{3} i + \frac{[4e^3 + 4]}{3} j + \frac{7K}{4}$$

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