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17/ENG04/057

ELECTRICAL ELECTRONICS ENGINEERING

ELECTROMAGNETICS EEE316

QUESTIONS (4&7)

Q 4. If the medium through which the em wave propagates is a good conductor,

(a) Analyse the equation in Q3 to show that the wave amplitude decreases exponentially as it penetrates the medium.

(b) Define the depth of penetration known as the skin depth and derive its value in terms of the parameters of the medium and the frequency of the signal.

(c) Calculate the depth of penetration of the wave in a sheet of copper at a frequency of 10 MHz at which the wave amplitude decreases to 1% of its value upon entering the sheet.

Take $\sigma = 5.8 \times 10^7$ S/m, $\mu_r = 1$ for copper.

Q7 (a) An air-filled coaxial transmission line has an outer conductor inside diameter, b = 10 mm and an inner conductor outside diameter, a = 3 mm. Calculate the,

(a) Capacitance per meter, C

(b) Inductance per meter, L

(c) Characteristic impedance, Z_0

(d) Phase velocity, v_p of an em wave propagate through it.

Hint: $C = \frac{2\pi\varepsilon_0}{\log_e \frac{b}{a}}$ $\quad L = \frac{\mu_0}{2\pi} \log_e \frac{b}{a}$

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17/ENG04/057
ELECTRICAL ELECTRONICS (ASSIGNMENT ON EEE 316)
ELECTROMAGNETICS FIELD AND WAVES

Question 4

a Analyse the equation in Question (3) to show that the wave amplitude decreases exponentially as it penetrates the medium?

4A

equation

$$\frac{\delta^2 E_y}{\delta x^2} = (j\omega\mu\sigma - \omega^2\mu\varepsilon)E_y$$

Analyzing

$$\frac{\delta^2 E_y}{\delta x^2} = (j\omega\mu\sigma - \omega^2\mu\varepsilon)E_y$$ ∵ An equation in em wave propagation in material medium.

$$\frac{\delta^2 E_y}{\delta x^2} = (j\omega\mu\sigma - \omega^2\mu\varepsilon)E_y \quad \text{eq(i)}$$

$$= \gamma^2 E_y \quad \text{eq(ii)}$$

Where; γ = Propagation constant

Note that:

Propagation in a good conducting medium - Neglect $\omega^2\mu\varepsilon$

∴ $\sigma >> \omega^2\mu\varepsilon$

eq (i) becomes

$$\frac{\delta^2 E_y}{\delta x^2} = (j\omega\mu\sigma)E_y$$

$$\therefore \gamma^2 = j\omega\mu\sigma$$

$$\gamma = \sqrt{j\omega\mu\sigma} \quad \text{eq(iii)}$$

Expressing γ as a complex parameter in terms of real and imaginary part;

$$\gamma = \alpha + j\beta \quad \text{eq(iv)}$$

α = Real $j\beta$ imaginary part

α = the attenuation constant in neper m^{-1}
β = the phase constant in radian m^{-1}

A solution of equation (ii) of a wave travelling in x direction is

$$E_y = E_0 e^{-\gamma x} = E_0 e^{-\alpha x} \cdot e^{-j\beta x} \quad \text{--- eqn (iv)}$$

from equation (iii)

$$\gamma^2 = j\omega\mu\sigma$$
$$\gamma = \sqrt{j\omega\mu\sigma}$$

∴ Recall that;

$$\sqrt{j} = \frac{1+j}{\sqrt{2}}$$

<u>Proving this:</u>

RHS;

$$\sqrt{j} \Rightarrow (\sqrt{j})^2 = \underline{\underline{j}}$$

LHS

$$\frac{1+j}{\sqrt{2}} = \left(\frac{1+j}{\sqrt{2}}\right)^2 = \frac{1+2j+j^2}{2} \qquad \text{where } (j^2) = -1$$

$$\therefore \frac{1+2j-1}{2} = \frac{2j}{2} = \underline{\underline{j}}$$

∴ it has been proved that $\sqrt{j} = \frac{1+j}{\sqrt{2}}$

using $\sqrt{j} = \frac{1+j}{\sqrt{2}}$ (Applying this to eq (iii))

$$\sqrt{j} = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

eqn (iii) becomes;

$$\gamma = \frac{1}{\sqrt{2}} \cdot \sqrt{\omega\mu\sigma} + \frac{1}{\sqrt{2}} \cdot j\sqrt{\omega\mu\sigma} \quad \text{--- eq (v)}$$

∴ Recall that; $\gamma = \alpha + j\beta$ ...

∴ Referring to equation (v).

$$\therefore \quad \alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \text{and} \quad \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \cdots \text{eqn (vi)}$$

Put eqn (vi) into eqn (iv)

$$\therefore E_y = E_0 e^{-\alpha x} \cdot e^{-j\beta x}$$

$$E_y \Rightarrow E_0 e^{-\sqrt{\frac{\omega\mu\sigma}{2}}x} \cdot e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}x} \quad \cdots \text{eqn (vii)}$$

Where δ; depth of peneration / skin depth $= \sqrt{\frac{2}{\omega\mu\sigma}}$

where $\omega = 2\pi f$

$$\therefore \delta = \sqrt{\frac{2}{2\pi f\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

equation (vii) becomes

$$E_y = E_0 e^{-x/\delta} e^{-jx/\delta}$$

∴ amplitude wave decreases exponentially as it penetrates into a conducting medium by a factor $e^{-x/\delta}$.

∴ When $x = \delta$; $e^{-x/\delta} = e^{-1}$

; waves decreases by e^{-1} = 0.37 of its value wh

entering the Conducting medium.

Representing using tabular form:

$\because \; E_y = E_0 e^{-x/\delta}$

$E_0 = 100$

$-x/\delta$	E_y
0	100
1	36.79
2	13.53
3	4.98
4	1.83
5	0.674
6	0.25
7	0.09
8	0.03
9	0.012
10	0.004

$\because \; E_y = 100 * e^{-0} = 100$

$E_y = 100 * e^{-1} = 36.79$

$E_y = 100 * e^{-2} = 13.53$

$E_y = 100 * e^{-3} = 4.98$

$E_y = 100 * e^{-4} = 1.83$

$E_y = 100 * e^{-5} = 0.674$

$E_y = 100 * e^{-6} = 0.25$

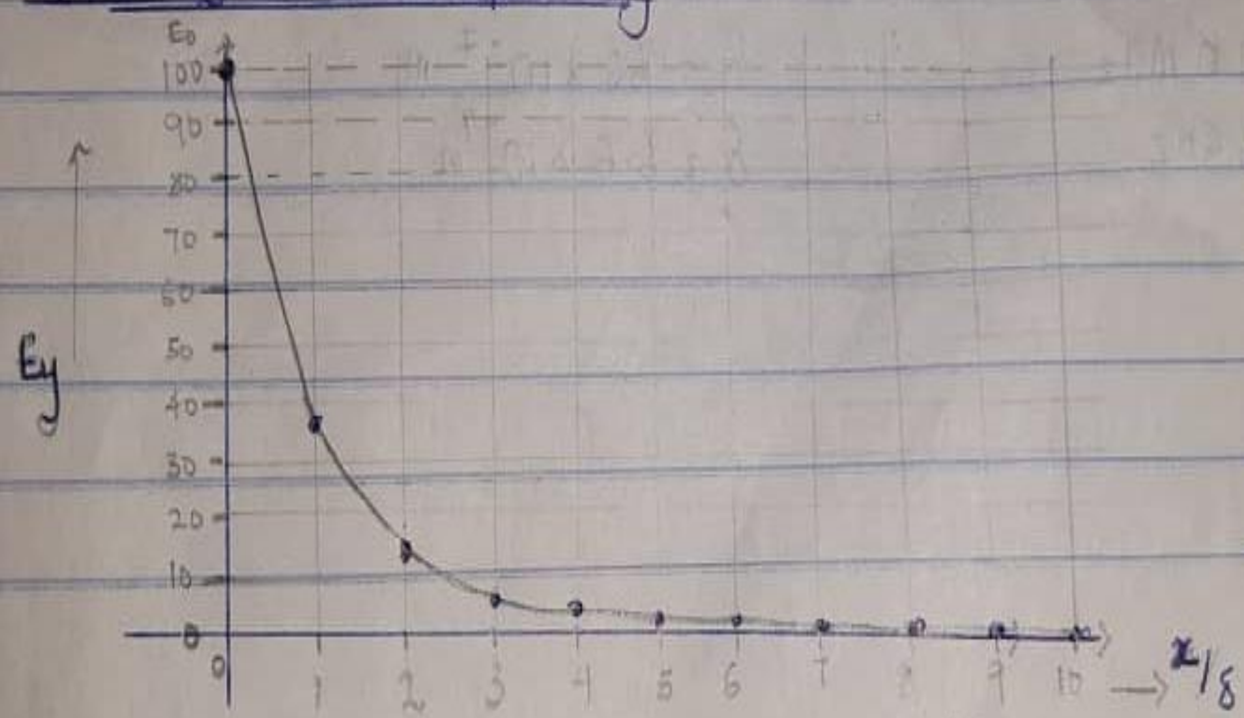
$E_y = 100 * e^{-7} = 0.09$

$E_y = 100 * e^{-8} = 0.03$

$E_y = 100 * e^{-9} = 0.012$

$E_y = 100 * e^{-10} = 0.004$

Representing this Graphically:



Wave Attenuation in a Conducting Medium

4B

b) Define the depth of penetration / Skin depth and derive its value in terms of parameters of the medium and the frequency of the signal

Depth of penetration / Skindepth :

This is the distance at which the wave reduces or decays to 37% of E_0 when moving from medium 1 to medium 2. Skin depth is dependent on the frequency, the higher the frequency the lower the skin depth

∴ It is represented with symbol δ

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$$

∴ μ_0 = Permeability of medium Hm^{-1}
f = frequency in Hz
σ = Conductivity of medium in $\mho m^{-1}$

Deriving its values at different frequency.

For copper $\mu_r = 1$, $\mu_0 = 4\pi \times 10^{-7}$, $\sigma = 58 M \mho m^{-1}$

$$\therefore \quad \delta = \frac{6.6 \times 10^{-2}}{\sqrt{f}}$$

∴ at specific frequencies

at 60 Hz $\quad \delta = 8.5 \times 10^{-3} m$
at 1 MHz $\quad \delta = 6.6 \times 10^{-5} m$
at 30 GHz $\quad \delta = 3.8 \times 10^{-7} m$
at 100 MHz $\quad \delta = 66 \times 10^{-7} m$
at 10 GHz $\quad \delta = 6.6 \times 10^{-7} m$

c)

c) Calculate the depth of penetration:

Preamble

$f = 10\,MHz$

$\sigma = 5.8 \times 10^{7}$

$\mu_r = 1$ for copper

$\mu_o = 4\pi \times 10^{-7}$

Using the formula;

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\delta = \frac{1}{\sqrt{\pi \times 10 \times 10^{6} \times 5.8 \times 10^{7} \times 4\pi \times 10^{-7}}}$$

$$\delta = 2.09 \times 10^{-5} m$$

Upon entering the sheet depth decreases by $1\% = 10^{-2}$.

$\therefore$

$$e^{-x/\delta} = 10^{-2}$$

$$\ln(e^{-x/\delta}) = \ln(10^{-2})$$

$$-x/\delta = -4.61$$

$$-x = -4.6\,\delta$$

$$x = 4.6\,\delta$$

$\therefore$ Where; $\delta = 2.09 \times 10^{-5}$

$$x = 4.6 \times 2.09 \times 10^{-5}$$

$$x = 9.6 \times 10^{-5} m.$$

$\therefore$ 1% of depth $= 9.6 \times 10^{-5} m$ (percentage depth of pen

Question (7).

An air filled co-axial transmission line has an outer conductor inside diameter, $b = 10mm$ and inner conductor outer diameter, $a = 3mm$ Calculate

(a) Capacitance, Inductance, Impedance, Phase velocity.

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Solution

Preamble:

a = Outside radius of inner conductor

$\therefore a = 3mm$ diameter

$a = 1.5mm$ radius.

b = Inside radius of outer conductor

$b = 10mm$ diameter = ~~[illegible]~~

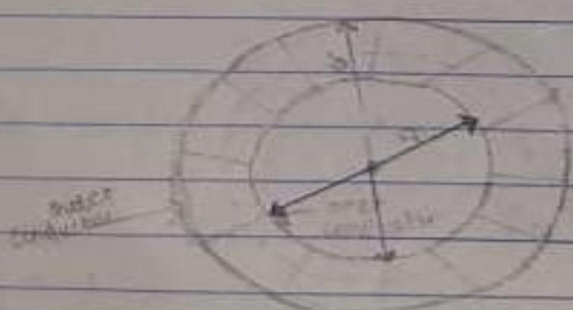
$b = 5mm$ radius

$\epsilon_r = 1$ for an air filled

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m

Diagramatic Representation.



(a) Capacitance;

diameter $a = 3mm \Rightarrow 0.003m$

$b = 10mm \Rightarrow 0.01m$

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using the formula; $C = \dfrac{2\pi\epsilon_0}{\log_e b/a}$

$$C = \frac{2 \times \pi \times 8.85 \times 10^{-12}}{\log_e \frac{0.01}{0.003}}$$

$$C = 4.62 \times 10^{-11}$$

$C \simeq$ 46 pF/meter

$L \simeq$ 46 pF/meter

(B) Inductance

$$L = \frac{\mu_0}{2\pi} \log_e \frac{b}{a}$$ 7b

$$L = \frac{4\pi \times 10^{-7}}{2\pi} * \log_e \left(\frac{0.01}{0.003} \right)$$

$L = 2.4 \times 10^{-7}$ Henry/m

(C) Characteristic Impedance, Z_0

$$Z_0 = \sqrt{\frac{L}{C}}$$ 7c

$$= \sqrt{\frac{2.4 \times 10^{-7}}{46 \times 10^{-12}}}$$

$= 72.23\ \Omega$

d) Phase velocity

$$V = \frac{1}{\sqrt{LC}}$$ 7d

$$V = \frac{1}{\sqrt{2.4 \times 10^{-7} \times 46 \times 10^{-12}}}$$

$V = 30.10 \times 10^{7}$ ms^{-1}