19/ENG05/054 MECHATRONICS ENGINEERING

OMOLUABI SEAN SEREMI

MAT 104

1a)

1b) $y = 4/x^3 \Rightarrow y = 4x^{-3}$

$y + \Delta y = \dfrac{4}{(x+\Delta x)^3}$

$\Delta y = \dfrac{4}{(x+\Delta x)^3} - \dfrac{4}{x^3}$

$\dfrac{\Delta y}{\Delta x} = \dfrac{4x^3 - 4(x+\Delta x)^3}{\Delta x \cdot x^3 (x+\Delta x)^3} = \dfrac{4x^3 - 4(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3)}{x^6\Delta x + 3x^5\Delta x^2 + 3x^4\Delta x^3 + x^3\Delta x^4}$

$= \dfrac{-12x^2\cancel{\Delta x} - 12x\cancel{\Delta x}^2 - 12\cancel{\Delta x}^3}{x^6\cancel{\Delta x} + 3x^5\cancel{\Delta x}^2 + 3x^4\cancel{\Delta x}^3 + x^3\Delta x^4} = \dfrac{-12x^2 - 12x\Delta x - 12\Delta x^2}{x^6 + 3x^5\Delta x^2 + 3x^4\Delta x^3 + x^3\Delta x^4}$

$\lim_{\Delta x \to 0} \dfrac{-12x^2 - 12x\Delta x - 12\Delta x^2}{x^6 + 3x^5\Delta x^2 + 3x^4\Delta x^3 + x^3\Delta x^4} = \dfrac{-12x^2}{x^6} = -12x^{-4}$ //

2a) $\displaystyle\int \frac{dx}{(x^2+36)} = \int \frac{dx}{(x^2+6^2)}$

let $x = 6\tan\theta$, $x^2 = 6^2\tan^2\theta$, $\theta = \tan^{-1}(x/6)$

$dx/d\theta = 6\sec^2\theta$, $dx = 6\sec^2\theta \cdot d\theta$

$\therefore \displaystyle\int \frac{6\sec^2\theta \cdot d\theta}{(6^2\tan^2\theta + 6^2)} = \int \frac{6\sec^2\theta \cdot d\theta}{6^2(\tan^2\theta + 1)}$

$\tan^2\theta + 1 = \sec^2\theta$

$\therefore \displaystyle\int \frac{6\cancel{\sec^2\theta} \cdot d\theta}{36\cancel{\sec^2\theta}} = \frac{6}{36}\theta + c = \frac{1}{6}\left[\tan^{-1}(x/6)\right] + C$ //

2b) $\displaystyle\int \frac{dx}{(x^2+13)} = \int \frac{dx}{(x+\sqrt{13})(x-\sqrt{13})} = \int \frac{A}{(x+\sqrt{13})} + \int \frac{B}{(x-\sqrt{13})}$

$\therefore 1 = A(x-\sqrt{13}) + B(x+\sqrt{13})$

let $x = -\sqrt{13}$

$1 = A(-2\sqrt{13})$

$A = \dfrac{1}{-2\sqrt{13}}$

let $x = \sqrt{13}$

$1 = B(2\sqrt{13})$

$B = \dfrac{1}{2\sqrt{13}}$

$\therefore \displaystyle\int \frac{dx}{(x^2+13)} = \int \frac{1}{2\sqrt{13}(x+\sqrt{13})} + \int \frac{1}{2\sqrt{13}(x-\sqrt{13})}$

$= -\frac{1}{2\sqrt{13}}\ln(x+\sqrt{3}) + \frac{1}{2\sqrt{13}}\ln(x-\sqrt{13}) + C$ //