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19/ENG05/048
MECHATRONICS ENGINEERING
MAT 102 ASSIGNMENT
~~Elijah~~

(1) If $A = 5i - 7j - 6K$, $B = j + 4k$, $C = 9i - 4j + K$ find $-8(A+B)\cdot(C-A)$

$A + B = (5i - 7j - 6K) + (j + 4k)$
$= (5i) + (-7j + j) + (-6K + 4K)$
$A + B = 5i - 6j - 2K$
$-8(A+B) = -8(5i - 6j - 2K)$
$= -40i + 48j + 16K$
$C - A = (9i - 4j + K) - (5i - 7j - 6K)$
$= (9i - 5i) + [-4j - (-7j)] + [K - (-6K)]$
$C - A = 4i + 3j + 7K$
$\therefore -8(A+B)\cdot(C-A) = (-40i + 48j + 16K)\cdot(4i + 3j + 7K)$
$= -160 + 144 + 112$
$= -160 + 256$
$\therefore -8(A+B)\cdot(C-A) = 96$

(2) Find a Unit Vector tangent to the space curve $x = -3t$, $y = t^2$, $z = 4t^3$ at the point where $t = 1$

$r = xi + yj + zK$
$\therefore r = -3ti + t^2 j + 4t^3 K$
$dr/dt = -3i + 2tj + 12t^2 K$
at $t = 1$
$dr/dt = -3i + 2j + 12K$
$\left|dr/dt\right|_{t=1} = \sqrt{(-3)^2 + 2^2 + 12^2} = \sqrt{9+4+144} = \sqrt{157} = 12.53$

~~Unit Tangent Vector (T)~~ $= \dfrac{dr/dt}{|dr/dt|}$

$\therefore T = \frac{-3i + 2j + 12k}{12.53}$ or $\frac{-3i}{12.53} + \frac{2j}{12.53} + \frac{12k}{12.53}$

(3) A particle moves along a curve $x = 8t^2$, $y = t^2 - 4t$, $z = t + 1$ where t is time; find the Acceleration.

$r = xi + yj + zk$

$\therefore r = 8t^2 i + (t^2 - 4t)j + (t+1)k$

Velocity $= \frac{dr}{dt}$; where r = displacement

$\therefore$ velocity $= \frac{dr}{dt} = 16ti + (2t-4)j + k$

Acceleration $= \frac{d^2 r}{dt^2} = 16i + 2j + k$

$\therefore$ Acceleration $= 16i + 2j + k$

$\therefore$ Acceleration $= 16i + 2j + k$

(4) If $A = i + 2j - 4k$, $B = 2i - 3j + k$, $C = 4j - 3k$ find $[(A \times B) \times C]$

$$\bar{A} \times \bar{B} = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 2 & -3 & 1 \end{vmatrix} = i\begin{bmatrix} 2 & -4 \\ -3 & 1 \end{bmatrix} - j\begin{bmatrix} 1 & -4 \\ 2 & 1 \end{bmatrix} + k\begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}$$

$$= i[2-12] - j[1-(-8)] + k[-3-4]$$

$$\bar{A} \times \bar{B} = -10i - 9j - 7k$$

$$\therefore [(\bar{A} \times B) \times \bar{C}] = \begin{vmatrix} i & j & k \\ -10 & -9 & -7 \\ 0 & 4 & -3 \end{vmatrix} = i\begin{bmatrix} -9 & -7 \\ 4 & -3 \end{bmatrix} - j\begin{bmatrix} -10 & -7 \\ 0 & -3 \end{bmatrix} + k\begin{bmatrix} 10 & -9 \\ 0 & 4 \end{bmatrix}$$

$$= i[27-(-28)] - j[30-0] + k[40-0]$$

$$[(A \times B) \times C] = 55i - 30j + 40k$$

(5) Given $R = 4\sin 3t\, i + 4e^{3t} j + 7t^3 K$; find the Integral of R with respect to t from 0 to 1

$$\int_0^1 R\,dt = \int_0^1 4\sin 3t\, i + \int_0^1 4e^{3t} j + \int_0^1 7t^3 K$$

$$= \left[4(-3\cos 3t)\, i\right]_0^1 + \left[4\left(\tfrac{1}{3}e^{3t}\right) j\right]_0^1 + \left[7\left(\tfrac{t^4}{4}\right) K\right]_0^1$$

$$= \left[-12\cos 3t\, i\right]_0^1 + \left[\tfrac{4}{3} e^{3t} j\right]_0^1 + \left[\tfrac{7}{4} t^4 K\right]_0^1$$

$$= \left[(-12\cos 3(1)) - (-12\cos 3(0))\right] i + \left[\left(\tfrac{4}{3}e^{3(1)} - \tfrac{4}{3}e^{3(0)}\right)\right] j + \left[\tfrac{7}{4}(1)^4 - \tfrac{7}{4}(0)^4\right] K$$

$$= [-11.98 - (-12)]\, i + [26.78 - 1.33]\, j + [7/4 - 0]\, K$$

$$= [-11.98 + 12]\, i + [25.45]\, j + [1.75]\, K$$

$$\therefore \int_0^1 R\,dt = 0.02i + 25.45j + 1.75K$$