Collins Sarima Lennenda
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1a $y = \sin \frac{3}{x^2}$

$u = \frac{3}{x^2} = 3x^{-2}$

$du/dx = -6x^{-3}$

$y = \sin u$

$\frac{dy}{du} = \cos u$

$dy/dx = \frac{du}{dx} \times \frac{dy}{du}$

$= -6x^{-3} \times \cos u$

$= -6\cos u x^{-3}$

Recall $u = \frac{3}{x^2}$

$= \frac{-6\cos \frac{3}{x^2}}{x^3}$

b $y = 4/x^3$

$y + \Delta y = \frac{4}{(x+\Delta 3)^3}$

$y + \Delta y = \frac{4}{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3}$

$\Delta y = \frac{4}{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3} - y$

$\Delta y = \frac{4}{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3} - \frac{4}{x^3}$

$= \frac{4\cancel{x}^3 - 4\cancel{x}^3 - 12x^2\Delta x - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 (x^3)}$

$= \frac{-12x^2\Delta x - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 (x^3)}$

$\frac{\Delta y}{\Delta x} = \frac{-12x^2\Delta x - 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 (x^3)} \times \frac{1}{\Delta x}$

$= \frac{-12x^2 - 12x\Delta x - 4(\Delta x)^2}{x^6 + 3x^5(\Delta x) + 3x^4 + x^3(\Delta x)^3}$

$$\lim_{\Delta x \to 0} \frac{dy}{dx} = \frac{-12x^2 - 12x(0) - 4(0)}{x^6 + 3x^5(0) + 3x^4(0) + x^3(0)}$$

$$= \frac{-12x^2}{x^6}$$

$$= \frac{-12}{x^4}$$

2a $$\int \frac{dx}{x^2 + 36} = \int \frac{1}{x^2 + 36} \, dx$$

$u = \frac{x}{6}$

$du = \frac{1}{6} dx$

$6du = dx$

$$\int \frac{1}{36\left(\frac{x^2}{36} + 1\right)} \, dx$$

$$\frac{1}{36} \int \frac{1}{\left(\frac{x}{6}\right)^2 + 1} \, dx$$

$$= \frac{1}{36} \int \frac{1}{u^2} + 1 \; 6du$$

$$= \frac{1}{36} \cdot \frac{6}{1} \int \frac{1}{u^2 + 1} \, du$$

$$= \frac{1}{6} \int \frac{1}{u^2 + 1} \, du$$

$$= \frac{1}{6} \tan^{-1} u$$

$$= \frac{1}{6} \tan^{-1} \frac{x}{6} + c$$

2b $\int \frac{dx}{x^2+13} = \int \frac{1}{x^2+13}\, dx$

$\int \frac{1}{13\left(\frac{x^2}{13}+1\right)}\, dx$

$u = \frac{x}{\sqrt{13}}$

$du = \frac{1}{\sqrt{13}}\, dx$

$\sqrt{13}\, du = dx$

$= \frac{1}{13} \int \frac{1}{\left(\frac{x}{\sqrt{13}}\right)^2+1}\, dx$

$= \frac{1}{13} \int \frac{1}{(u)^2+1} \sqrt{13}\, du$

$= \frac{1}{13} \cdot \sqrt{13} \int \frac{1}{u^2+1}\, du$

$= \frac{1}{\sqrt{13}} \int \frac{1}{u^2+1}\, du = \frac{1}{\sqrt{30}} \tan^{-1} u$

$= \frac{1}{\sqrt{30}} \tan^{-1} \frac{x}{\sqrt{30}} + C$