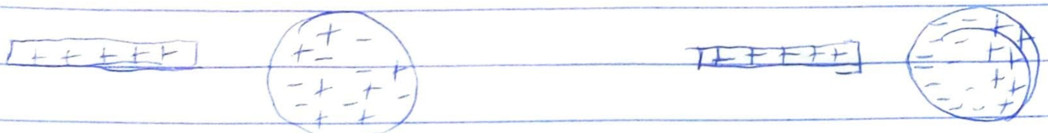
NAME: OKWUOKWU BRYAN COURSE: PHY 102
MATRIC NO: 19/ENG03/049 DEPT: Mechatronics

1 (a) Charging by induction

Consider a ~~negatively~~ positively charged rubber rod brought near a neutral conduction sphere, insulated without any conducting path to ground as below. The repulsive force between the ~~electrons~~ positive charges in the rod and those in the sphere cause those in the sphere to move away from the rod. The region near the rod has an excess of electrons. If a grounded conducting wire is then connected to the sphere some positive charges leave the sphere. If the wire is removed, the sphere is left with excess electrons.

Finally, when the rod is removed from the sphere, the positive charge is distributed evenly over the sphere's surface



1 (b) $K = 9 \times 10^9$ $\qquad q_1 + q_2 = 5 \times 10^{-5}\,C$

$F = 1\,N$ $\qquad r = 2\,m$

Recall, $\cancel{F = \frac{Kq_1q_2}{r^2}}$ $\qquad 1 = \frac{9 \times 10^9 \times (q_1(q_1 - 5 \times 10^{-5}))}{2^2}$

$4 = 9 \times 10^9 \times 5 \times 10^{-5} q_1 + 9 \times 10^9 q_1^2$

$4 = -4.5 \times 10^5 q_1 + 9 \times 10^9 q_2$

$9 \times 10^9 q_1^2 - 4.5 \times 10^5 q_1^2 - 4 = 0$

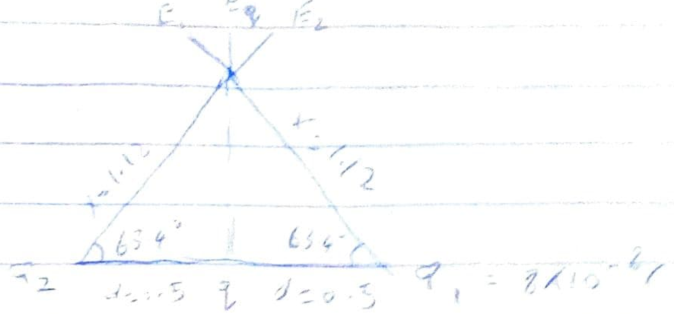
Solving the quadratic equation

$q_1 = 1.11 \times 10^{-5}\,C$ or $3.8 \times 10^{-5}\,C$

$q_2 = 1.11 \times 10^{-5}\,C$ or $3.8 \times 10^{-5}\,C$

E_q $q = Q_1 = 8\mu C$

$d = 0.5$



$r^2 = 1^2 + 0.5^2$ $\quad \sqrt{x^2} = \sqrt{1.25} \Rightarrow x = 1.12$

$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{1}{0.5} = 2 \quad \therefore \theta = \tan^{-1}(2) = 63.4°$

$E_1 = \frac{Kq_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 5739.\cancel{7}95918$

$E_2 = \frac{Kq_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 5739.795918$

$E_q = \frac{Kq}{r^2} = \frac{9 \times 10^9 \times q}{1} = 9 \times 10^9 q$

Vector	angle	x - component	y - component
$E_1 = 5739.795918$	$63.4°$	-2570.045	5132.262
$E_2 = 5739.795918$	63.4	2570.045	5132.262
$E_q = 9 \times 10^9 q$	$90°$	0	$9 \times 10^9 q$
		$\Sigma x = 0$	$\Sigma y = 10264.52568$

Magnitude $= \sqrt{(\Sigma x^2) + (\Sigma y^2)} \Rightarrow E_n = 0 = \sqrt{0^2 + (10264.52568)^2}$

$0 = 9 \times 10^9 q + 10264.52568$

$q = \frac{-10264.52568}{9 \times 10^9} = 11.4 \mu C$

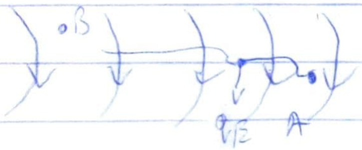
3) a) (i) Volume Charge Density, $\rho = \frac{dQ}{dV}$; $dQ = \rho dV$

(ii) Surface Charge density, $\sigma = \frac{dQ}{dA}$; $dQ = \sigma dA$

(iii) Linear Charge density, $\lambda = \frac{dQ}{dL}$; $dQ = \lambda dL$

3) b) Electric Potential Difference

The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in volts (V). It is a scalar quantity.



Consider the diagram above, suppose a test charge q_0 is moved from point A to point B along an arbitrary path inside a field E. The electric field exerts a force q_0E on the charge. To move the test charge from A to B at constant velocity, an external force of $-q_0E$ must act on the charge

$\therefore$ $dW = F \cdot dL$ -- (1)

But $F = -q_0E$ - (2) Putting (2) in (1)

$dW = -q_0 E dL$

Then total work done to move charge;

$$W(A \to B) = -q_0 \int_A^B E dL \quad - (4)$$

From the definition of electric potential difference; $V_B - V_A = \frac{W(A \to B)}{q_0}$

Putting (4) in (5); $V_B - V_A = -\int_A^B E dL$

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4(a) Magnetic flux is defined as the strength of the magnetic field which can be represented by lines of forces.

(b) $m = 9 \times 10^{-31}$ kg $r = 1.4 \times 10^{-7}$ m $B = 3.5 \times 10^{-1}$ weber/meter

Cyclotron frequency = angular speed => $W = \frac{V}{r} = \frac{qB}{m}$

$W = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}} = \cancel{6.2222}\ T^{-1}$ $6.22 \times 10^{10}\ T^{-1}$

(c) We are given; ① electron mass $= 9.11 \times 10^{-31}$ kg
② radius $= 1.4 \times 10^{-7}$ m
③ magnetic field $= 3.5 \times 10^{-1}$ weber/Meter Square

and asked to find the cyclotron frequency which is essentially angular speed.

Angular speed $= W = \frac{V}{r} = \frac{qB}{m}$

Putting in known values

$W = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}} = 6.22 \times 10^{10}\ T^{-1}$

And since cyclotron frequency is equal to angular speed;

Cyclotron frequency $= 6.22 \times 10^{10}\ T^{-1}$

5a) Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ), the current (I), the change in length, the radius and inversely proportional to square of radius (r^2)

Mathematically: $dB = \frac{\mu_0}{4\pi} \frac{I\, dL \times r}{r^2}$

5b) Magnetic field of a straight current carrying conductor

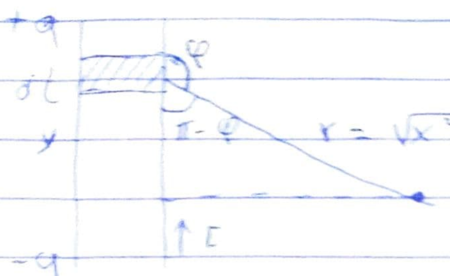

Fig 2; A section of a straight carrying conductor

Apply Biot-Savart law, we find magnitude of the field dB

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin\varphi}{r^2} \qquad \text{But } \sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad - (1)$$

But $\sin(\pi - \varphi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}}$ (2)

Put (2) in (1): $B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{3/2}} \quad (dl = dy)$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy$$

Using special integrals, $\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$

Hence: $B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2(x^2+y^2)^{1/2}} \right]_{-a}^{a}$

$$= \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

When length $2a$ is very great compared to its distance x from point P, we consider it infinitely long

$$(x^2+a^2)^{1/2} = a$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

But $x = r$

$$\therefore B = \frac{\mu_0 I}{2\pi r}$$

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