

Section A

3a) State the formulation of the following identities of charges.

- i) Volume Charge Density, $\rho = \frac{dq}{dv} \rightarrow dq = \rho dv$
- ii) Surface Charge Density, $\sigma = \frac{dq}{dA} \rightarrow dq = \sigma dA$
- iii) Linear Charge Density, $\lambda = \frac{dq}{dl} \rightarrow dq = \lambda dl$

3b) Explain with appropriate equations, the electric potential difference

An electric field E , exerts a force, $F = q_0 E$ on a charge. To move the test charge from points A to B at constant velocity, an external force of $F = q_0 E$ must act on the charge. Then, the work done, dw , is:

$$dw = F \cdot dl \dots \textcircled{1}$$

$$F = q_0 E \dots \textcircled{2}$$

Substituting equation 2 in 1;

$$dw = -q_0 E dl \dots \textcircled{3}$$

The total work done in moving the charge from point A to B is:

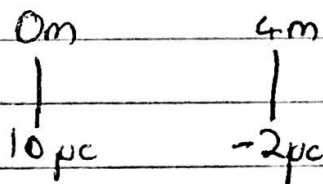
$$W(A \rightarrow B)_{Aq} = -q_0 \int_A^B E dl \dots \textcircled{4}$$

From electric potential difference,

$$V_B - V_A = \frac{W(A \rightarrow B)_{Aq}}{q_0} \dots \textcircled{5}$$

From equation 4 and 5, we have: $V_B - V_A = - \int_A^B E dl \dots \textcircled{6}$

3c) Two point charges $Q_1 = 10 \mu C$ and $Q_2 = -2 \mu C$ are arranged along the x -axis at $x = 0$ and $x = 4m$ respectively. Find the position along the x -axis where $v = 0$.



$$F = \frac{k q_1 q_2}{r^2}$$

$$q_1 = 10 \times 10^{-6} C, q_2 = -2 \times 10^{-6} C, r = 4m, k = 9 \times 10^9$$

$$F = \frac{9 \times 10^9 \times 10 \times 10^{-6} \times -2 \times 10^{-6}}{4^2}$$

$$F = 0.0113 \text{ N}$$

$$W = F \times d = 0.0113 \times 4 = 0.045 \text{ J}$$

Work done at the position of 4m = -0.045 J

$$\therefore V = W/q = -0.045 / -2 \times 10^{-6} = 22500 \text{ volts}$$

Work done at the position of 10m = $0.0113 \times 0 = 0 \text{ J}$

$$\therefore V \text{ at the position of } 10\text{m} = 0$$

$$\text{Ans} = \underline{\underline{10\text{m}}}$$

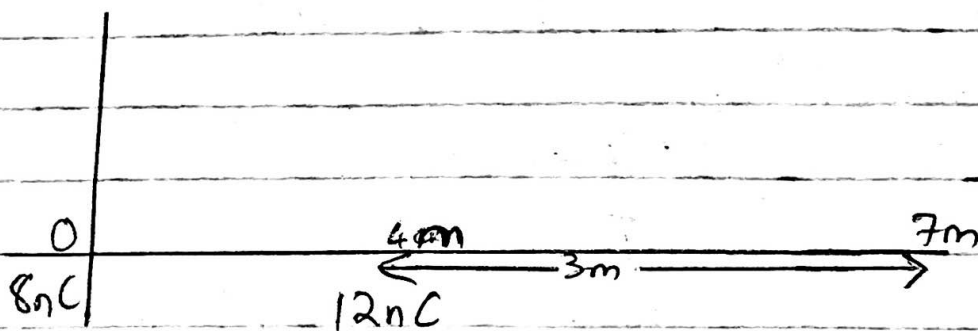
2a). Distinguish between the terms: Electric Field and Electric Field Intensity.

Solution

An electric field is a region of space in which an electric charge will experience an electric force while electric field intensity is the measure of the strength of an electric field at any point.

2b). A positive charge $Q_1 = 8 \text{ nC}$ is at the origin and a second positive charge $Q_2 = 12 \text{ nC}$ is on the x -axis at $x = 4 \text{ m}$. Find;

i). The net electric field at a point P on the x axis at $x = 7 \text{ m}$



$$\text{Electric field} = \Sigma = \frac{F}{q} = \frac{kq}{r^2}$$

$$k = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}, \quad q = 12 \times 10^{-9} \text{ C}, \quad r = 3 \text{ m}; \quad q = 8 \times 10^{-9} \text{ C}; \quad r_2 = 7 \text{ m}$$

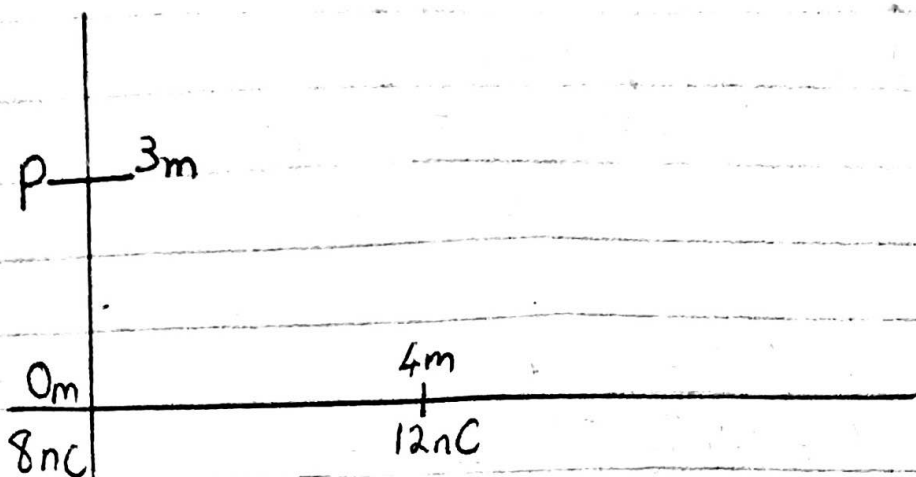
$$\Sigma_1 = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{3^2} = 12 \text{ N/C}$$

$$\therefore \Sigma_1 = 12 \text{ N/C}$$

$$\Sigma_2 = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2} = 1.47$$

$$\Sigma_{\text{net}} = 1.47 + 12 = 13.47 \text{ N/C}$$

ii) the electric field at a point Q on the y axis at $y = 3\text{m}$ due to the charges



$$Q_1 = 8 \times 10^{-9} \text{ C}, Q_2 = 12 \times 10^{-9}, r_1 = 3\text{m}, r_2 = 7\text{m}, k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$E_1 = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \text{ N/C}$$

$$E_2 = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{7^2} = 2.204 \text{ N/C}$$

$$E_{\text{net}} = (8 + 2.204) \text{ N/C} = 10.204 \text{ N/C}$$

Section B.

4a) What is Magnetic Flux?

Magnetic Flux is defined as the strength of magnetic field represented by lines of force.

$$4b) m = 9.11 \times 10^{-31} \text{ kg}, q = 1.6 \times 10^{-19} \text{ C}, r = 1.4 \times 10^{-7} \text{ m}, B = 3.5 \times 10^{-1} \text{ weber/m}^2$$

$$\omega = \frac{qB}{m} = \frac{1.60 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$\omega = 6.147 \times 10^{10} \text{ rad/s}$$

4c). The cyclotron frequency also known as the angular speed of an electron with mass $9.11 \times 10^{-31} \text{ kg}$ and radius $1.4 \times 10^{-7} \text{ m}$ with a magnetic field of $3.5 \times 10^{-1} \text{ weber/meter square}$ is given as $6.147 \times 10^{10} \text{ rad/s}$

5a) Biot - Savart law states that the magnetic flux density near a long, straight conductor is directly proportional to the current I in the conductor and inversely proportional to the distance from the conductor.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^2}$$

5b) Applying the Biot - Savart law, we find the magnitude of \vec{dB}

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin\theta}{r^2}$$

$$\sin(\pi - \theta) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{r^2}$$

$$r^2 = x^2 + y^2 \text{ (Pythagoras theorem)}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{x^2 + y^2} \dots \textcircled{i}$$

$$\text{But } \sin(\pi - \theta) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \dots \textcircled{ii}$$

Substituting equation ii into i we have,

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

$$\text{Recall } dl = dy, \quad B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \dots \textcircled{iii}$$

Using Special Integrals, $\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$

Equation (iii) becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2 + a^2)^{1/2}} \right)$$

When the length $2a$ of conductor is very great in comparison to its distance, x , from point P , we consider it infinitely long. That is when a is much larger than x .

$$(\sqrt{x^2 + a^2})^{1/2}$$

$$(\sqrt{x^2 + a^2})^{1/2} \approx a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$