Name: Oyedepo Oluwafikayomi Deborah
Department: Medicine and Surgery
Matric Number: 19/MHS 01/373

Course code: PHY 102

Course Title: Electricity, Magnetism and Modern Physics

COVID-19 HOLIDAY ASSIGNMENT

1a. Electrostatic Induction is the process of charging an object without touching it. Consider a negatively charged rubber rod brought near a neutral conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the ~~rod~~ electrons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some electrons move to the side of the sphere farthest away from the rod (fig. a). The region of the sphere nearest the negatively charged rod has an excess of positive charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere (as in fig. b), some of the electrons leave the sphere and travel to the earth. If the wire to ground is then removed (fig. c), the conducting sphere is left with an excess of induced positive charge. Finally, when the rubber rod is removed from the vicinity of the sphere (fig. d), the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere as shown below:

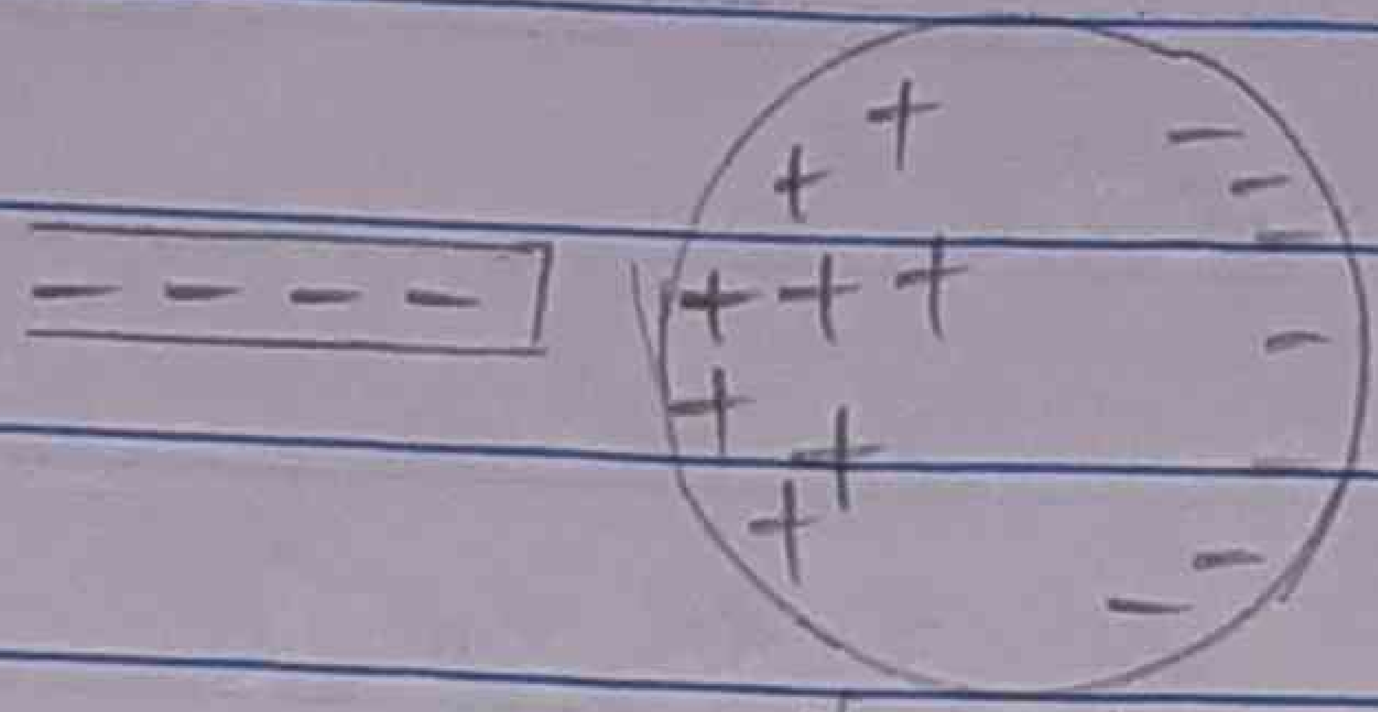


fig (a)

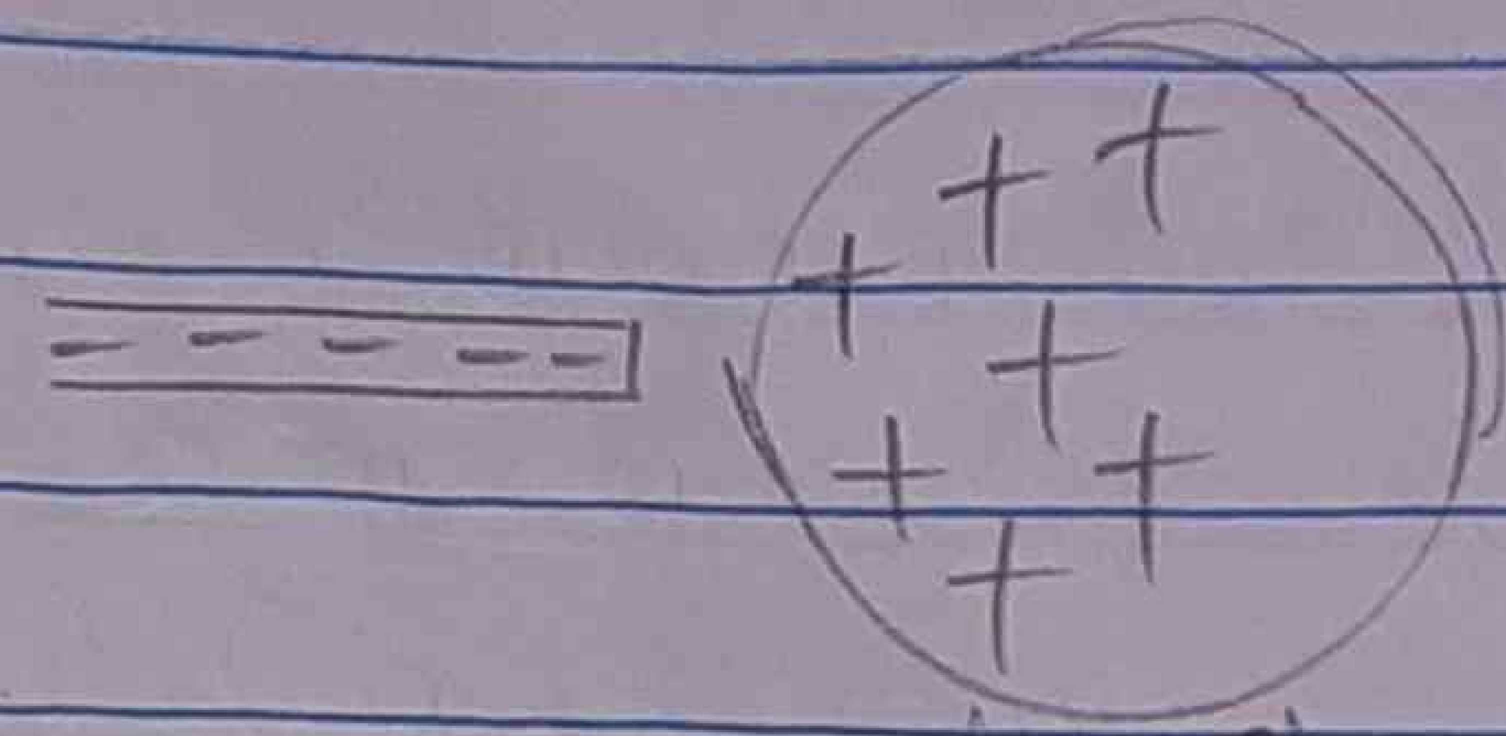


fig (c)

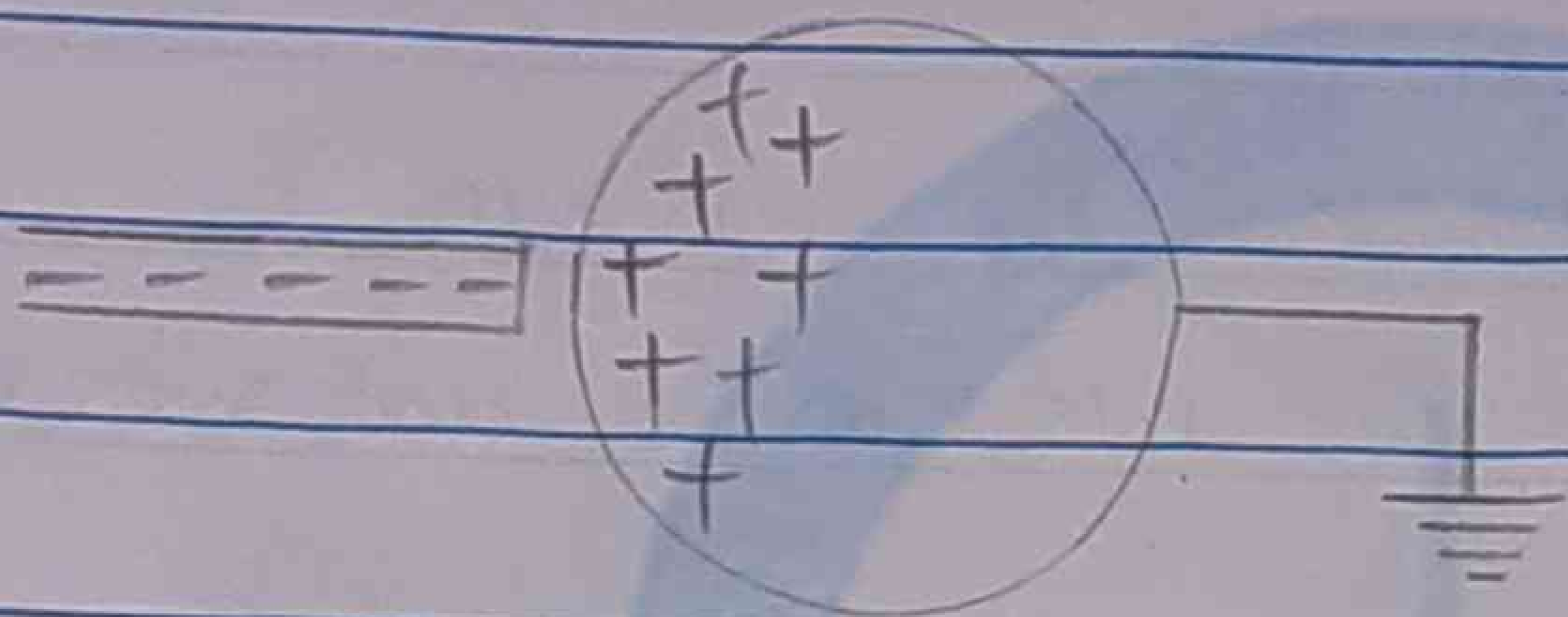


fig (b)

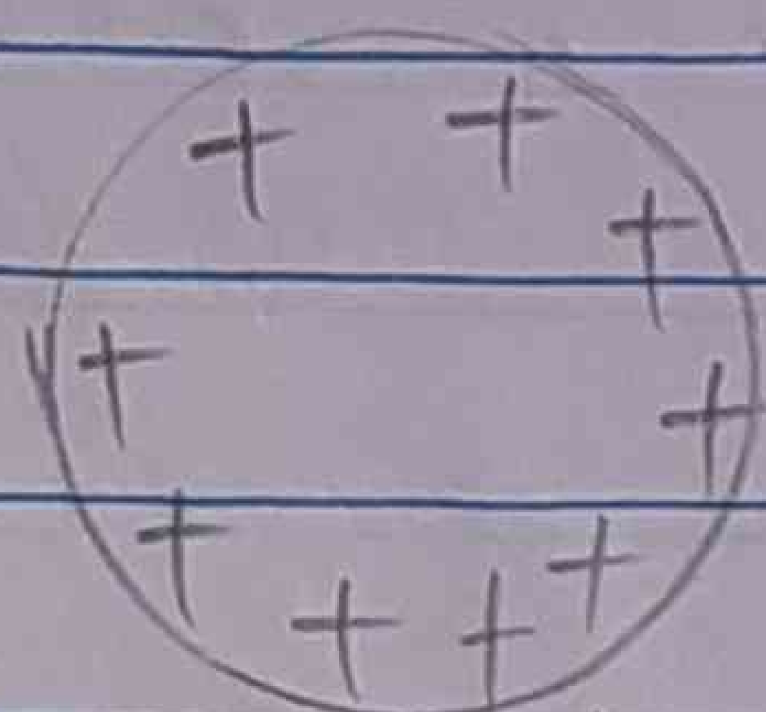


fig (d)

1b. $q_1 + q_2 = 5.0 \times 10^{-5} C$ —— ①

$F = 1.0 N$

$r = 2.0 m$; $k = 9 \times 10^9 \ Nm^2/C^2$

$F = \dfrac{k q_1 q_2}{r^2}$

$1 = \dfrac{9 \times 10^9 \times q_1 q_2}{2^2}$

$q_1 q_2 = \dfrac{4}{9 \times 10^9} = 4.44 \times 10^{-10} C^2$ —— ②

From eqtn ①, $q_2 = 5 \times 10^{-5} - q_1$ —— ③

Put eqtn ③ into ②

$q_1(5 \times 10^{-5} - q_1) = 4.44 \times 10^{-10}$

$5 \times 10^{-5} q_1 - q_1^2 = 4.44 \times 10^{-10}$

$q_1^2 - 5 \times 10^{-5} q_1 + 4.44 \times 10^{-10} = 0$

$$q_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$q_1 = \frac{-(-5\times10^{-5}) \pm \sqrt{(-5\times10^{-5})^2 - 4(1)(4.44\times10^{-10})}}{2(1)}$$

$$q_1 = 1.15\times10^{-5}C \quad \text{or} \quad 3.85\times10^{-5}C$$

when $q_1 = 1.15\times10^{-5}C$

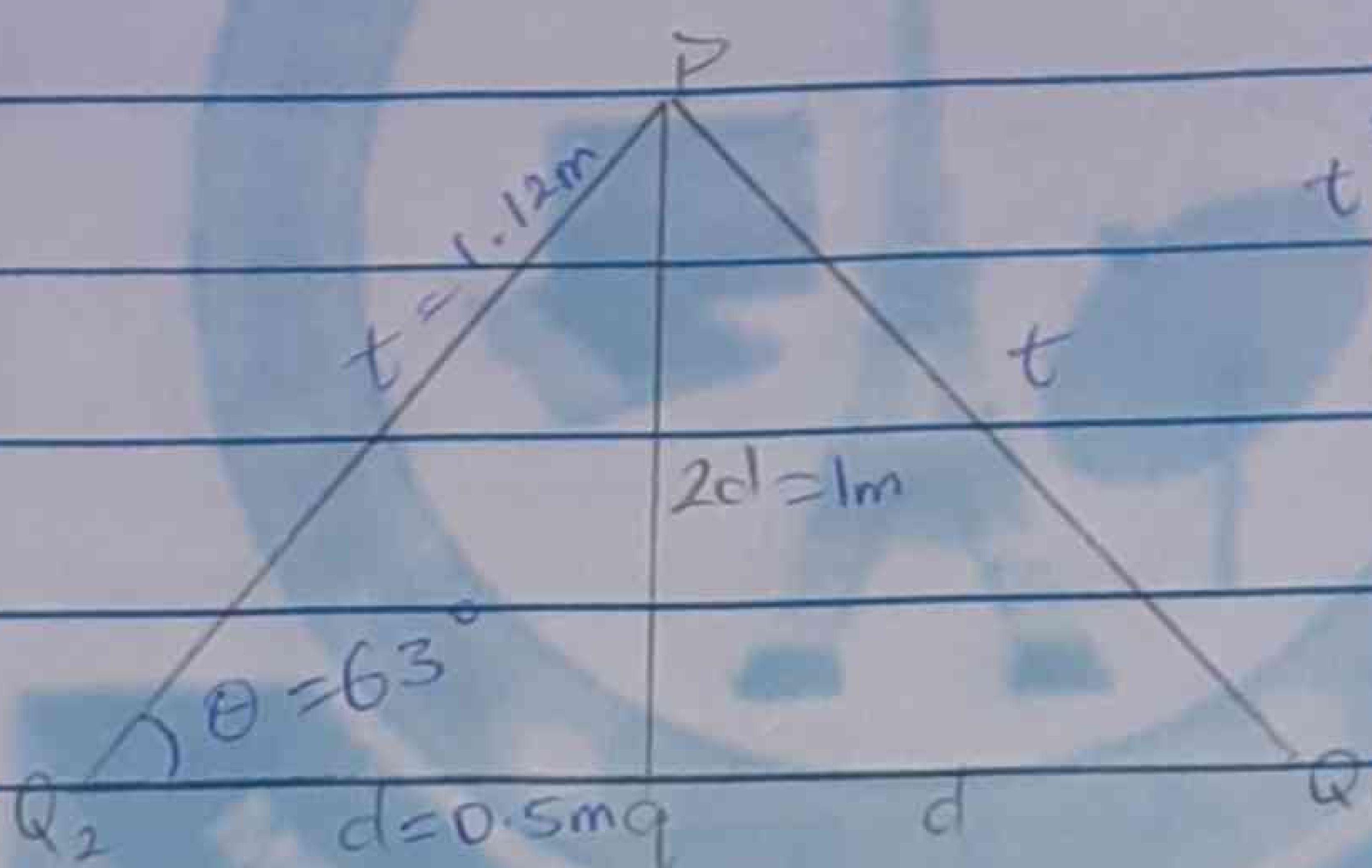
$$q_2 = (5\times10^{-5}) - (1.15\times10^{-5})C = 3.85\times10^{-5}C$$

when $q_1 = 3.85\times10^{-5}C$

$$q_2 = (5\times10^{-5}) - (3.85\times10^{-5})C = 1.15\times10^{-5}C$$

$\therefore$ The charges on the spheres are $1.15\times10^{-5}C$ and $3.85\times10^{-5}C$

1(c)



$$t^2 = 0.5^2 + 1^2$$

$$t^2 = 1.25$$

$$t = 1.12m$$

$$\tan\theta = \frac{2\cancel{d}}{\cancel{d}}$$

$$\tan\theta = 2$$

$$\theta = \tan^{-1}(2) = 63^\circ$$

$$E_1 = \frac{KQ_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.12)^2} = 57398\ N/C$$

$$E_2 = \frac{KQ_2}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.12)^2} = 57398\ N/C$$

$$E_q = \frac{kq}{r^2} = \frac{9\times10^9 \times q}{1^2} = 9\times10^9 q \ N/C$$

Vector	Angle	x-component	y-component
E_1	63°	$+E_1 \cos 63^\circ$	$-E_1 \sin 63^\circ$
		$= +57398 \cos 63^\circ$	$= -57398 \sin 63^\circ$
E_2	63°	$-E_2 \cos 63^\circ$	$-E_2 \sin 63^\circ$
		$= -57398 \cos 63^\circ$	$= -57398 \sin 63^\circ$
E_q	90°	$+9\times10^9 q \cos 90^\circ$	$-9\times10^9 q \sin 90^\circ$

$$E_x = +57398 \cos 63^\circ - 57398 \cos 63^\circ + 9\times10^9 q \cos 90^\circ$$
$$= 0 \ N/C$$

$$E_y = -57398 \sin 63^\circ - 57398 \sin 63^\circ - 9\times10^9 q \sin 90^\circ$$
$$E_y = -103316.4 - 9\times10^9 q$$
$$E_{net} = \sqrt{E_x^2 + E_y^2}$$
$$E_{net} = \sqrt{0^2 + (-103316.4 - 9\times10^9 q)^2}$$
$$E_{net} = -103316.4 - 9\times10^9 q$$

But E_{net} at point P is 0

$$-103316.4 - 9\times10^9 q = 0$$
$$\frac{9\times10^9 q}{9\times10^9} = \frac{-103316.4}{9\times10^9}$$

$$q = -1.14\times10^{-5} \ C$$

$$\therefore q = -11.4 \ \mu C$$

2a. An electric field is defined as a region of space in which an electric charge will experience an electric force while electric field intensity (also called electric field strength) is defined as the force per unit charge at a point in space. Electric field intensity is measured in Newton per coulomb (N/C)

b. $Q_1 = 8nC = 8 \times 10^{-9} C$

$Q_2 = 12nC = 12 \times 10^{-9} C$

(i)

$$E_1 = \frac{kq_1}{r_1^2} = \frac{8 \times 10^{-9} \times 9 \times 10^9}{7^2} = 1.47 N/C$$

$$E_2 = \frac{kq_2}{r_2^2} = \frac{12 \times 10^{-9} \times 9 \times 10^9}{3^2} = 12 N/C$$

Vector	Angle	x-comp	y-comp
E_1	$0°$	$E_1 \cos 0° = E_1$	$E_1 \sin 0° = 0$
E_2	$0°$	$E_2 \cos 0° = E_2$	$E_2 \sin 0° = 0$

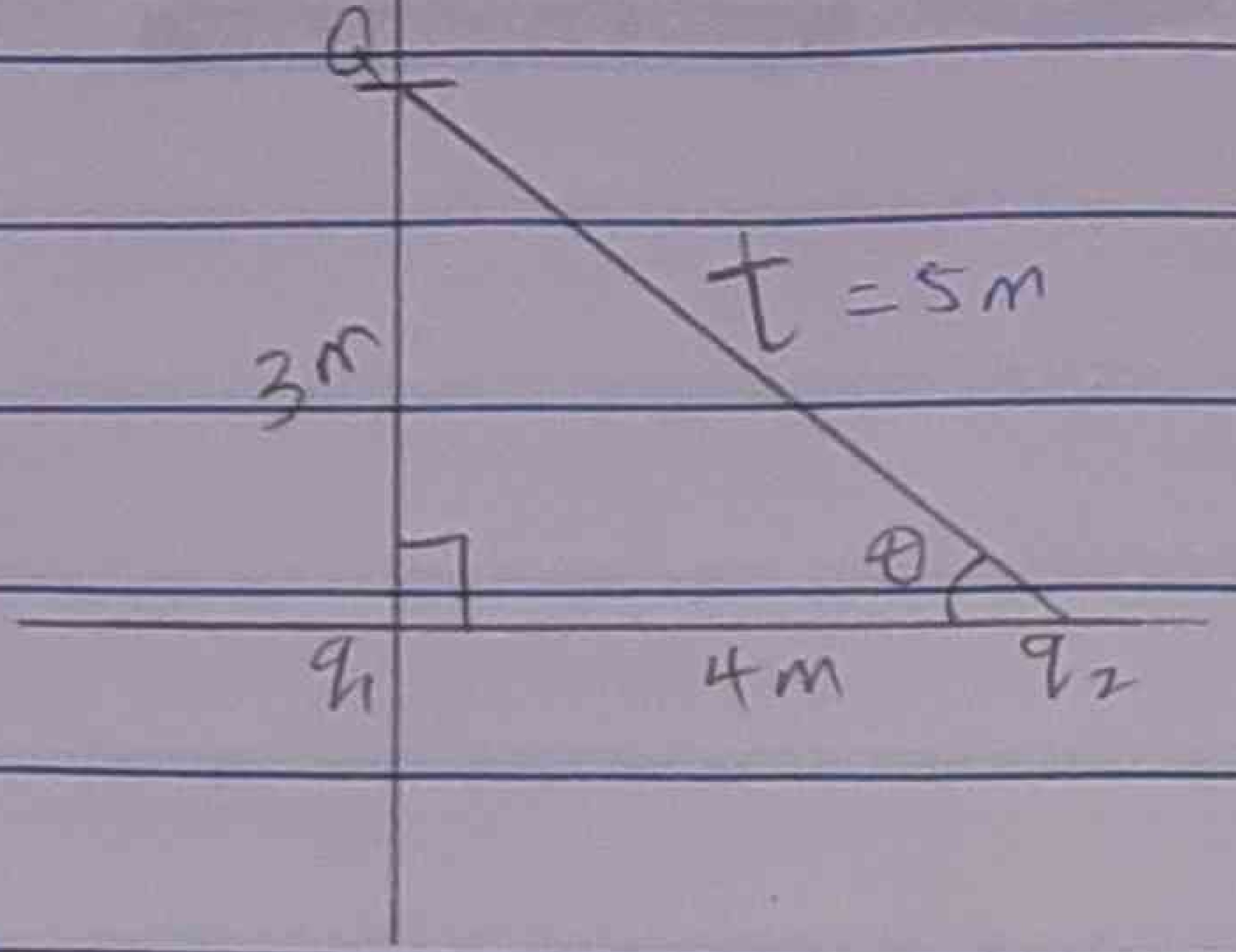
$E_x = E_1 + E_2 = (1.47 + 12) N/C = 13.47 N/C$

$E_y = 0 N/C$

$|E| = \sqrt{E_x^2 + E_y^2} = \sqrt{(13.47)^2 + 0^2} = 13.47 N/C$

$\therefore E_{net} = 13.47 N/C$

26/10



$t^2 = 3^2 + 4^2$

$t^2 = 9 + 16$

$t = \sqrt{25} = 5m$

$\tan\theta = 3/4$

$\theta = \tan^{-1}(3/4)$

$\theta = 36.87°$

$E_1 = \frac{Kq_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-9}}{3^2} = 8\,N/C$

$E_2 = \frac{Kq_2}{r^2} = \frac{9\times10^9 \times 12\times10^{-9}}{5^2} = 4.32\,N/C$

Vector	Angle	x-comp	y-comp
E_1	90°	$E_1 \cos 90° = 0$	$E_1 \sin 90° = E_1$
E_2	36.87°	$E_2 \cos 36.87°$ $= 0.8E_2$	$E_2 \sin 36.87° = 0.6E_2$

$E_x = 0 + 0.8E_2 = 0.8 \times 4.32 = 3.456\,N/C$

$E_y = \cancel{E_1 +} E_1 + 0.6E_2 = 8 + 2.592 = 10.592\,N/C$

$|E| = \sqrt{E_x^2 + E_y^2} = \sqrt{(3.456)^2 + (10.592)^2}$

$|E| = 11.14\,N/C$

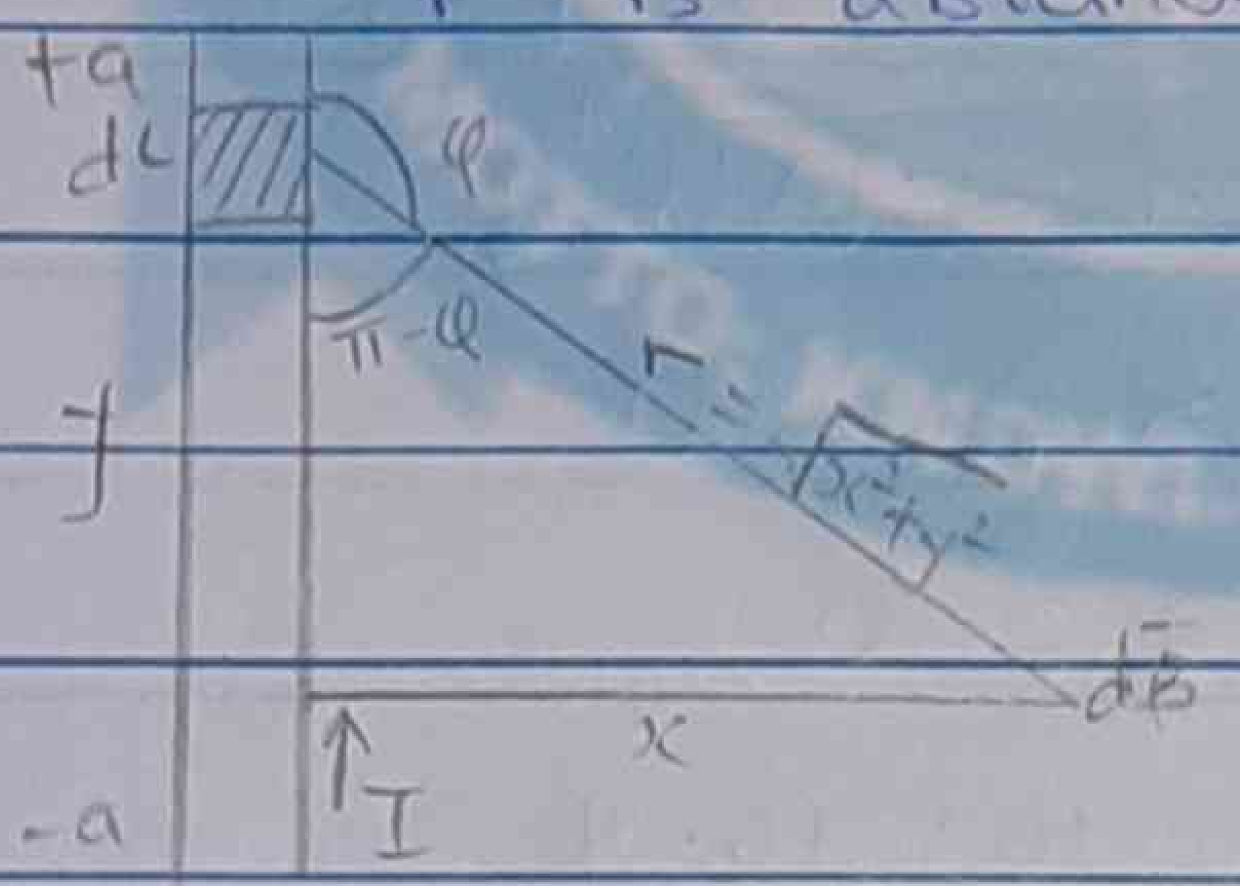
$\therefore E_{net} = 11.14\,N/C$

5a. Biot - Savart law states that the density of magnetic flux (dB) is directly proportional to the element length (dl), the ~~flow of current~~ (I), the sine of the angle θ among the flow of current direction and the vector combining a given position of the field ~~with the current~~ and inversely proportional to the square of the distance (r) of the specified point from the current element.

b.
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r} \times \sin\theta}{r^2} \quad (\sin 90^\circ = 1)$$

where μ_0 is permeability of free space
$d\vec{B}$ is magnetic field
I is current
$d\vec{l}$ is length element
r is distance

b.



Applying the Biot-Savart law,

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{dl \sin\theta}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin\varphi}{r^2}$$

$$\sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \sin(\pi - \varphi$$

$$\sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

From the diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad \text{——— (1)}$$

But from the diagram

$$\sin(\pi - \varphi) = \frac{x}{\sqrt{x^2+y^2}} = \Big(\cancel{\frac{x}{\sqrt{x^2+y^2}}}$$

$$\sin(\pi - \varphi) = \frac{x}{(x^2+y^2)^{1/2}} \quad \text{——— (2)}$$

Substitute eqtn (2) into (1)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \cdot \frac{x}{(x^2+y^2)^{1/2}(x^2+y^2)}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \cdot \frac{x}{(x^2+y^2)^{3/2}}$$

Recall that $dl = dy$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}}\, dy$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}}\, dy \quad \text{——— (3)}$$

using special integrals:

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\frac{y}{(x^2+y^2)^{1/2}}$$

Equation (3) becomes

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

when the length $2a$ of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long.

$$(x^2+a^2)^{1/2} \simeq a \; ; \; \text{as } a \to \infty$$

$$B = \frac{\mu_0 I}{2\cancel{4}\pi x}\left(\frac{\cancel{2}\cancel{a}}{\cancel{a}}\right)$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

6a. A practical application of the Faraday's law is the production of sound in an electric guitar. The coil in this case called the pickup coil is placed near the vibrating guitar string which is made of a metal that can be magnetized. A permanent magnet inside the coil magnetizes the portion of the string nearest the coil. When the string vibrates at some frequency, its magnetized segment produces a changing magnetic flux through the coil. The changing flux induces as emf in the coil that is fed to an amplifier. The output of the amplifier is sent to the loudspeakers, which produces the sound waves we hear.

b. $N = 300$ turns ; $R = 2\Omega$

$L = 10cm = 0.1m$ $A = L^2 = 0.1^2 = 0.01m^2$

$B_1 = 0T$ $B_2 = 10T$

$dt = 0.5 sec$

(i) $$|E| = N\frac{d\Phi_B}{dt} = N\frac{d(BA)}{dt}$$

$$|E| = N \cdot \frac{A(B_2 - B_1)}{dt} = NA\frac{dB}{dt}$$

$$|E| = 300 \times 0.01 \times \frac{10-0}{0.5} = 60V$$

$\therefore$ Magnitude of induced e.m.f $= 60V$

(ii) $E = IR$

$$I = \frac{E}{R} = \frac{60}{2} = 30A$$ $\therefore$ Magnitude of induced current $= 30A$

6c) $l \times B = 5\,cm \times 8\,cm = 0.05\,m \times 0.08\,m$

$A = 4 \times 10^{-3}\,m^2$

$N = 75$ turns

$R = 8\,\Omega$

$I = \cancel{0.8/8}\; 0.1\,A$

$|E| = IR$

$|E| = 0.1 \times 8 = 0.8\,V$

$$|E| = N \frac{d\Phi_B}{dt} = N \frac{d(BA)}{dt}$$

$$|E| = NA \frac{dB}{dt}$$

$$\frac{dB}{dt} = \frac{|E|}{NA}$$

$$\frac{dB}{dt} = \frac{0.8}{75 \times 4 \times 10^{-3}}$$

$$\frac{dB}{dt} = 2.67\,T/s$$

$\therefore$ Time rate of change of magnetic flux $= 2.67\,T/s$