

NAME: OBOLO FAITH IFEDOLWA

MAT NO: 19/MHSD/274

DEPT: MBBS

PHY 102 ASSIGNMENT - Answer 4 questions

Question 2a) Distinguish between the terms: electric field and electric field intensity.

2b) A positive charge  $Q_1 = 8\text{nC}$  is at the origin and a second positive charge  $Q_2 = 12\text{nC}$  is on the x-axis at  $x = 4\text{m}$ . Find

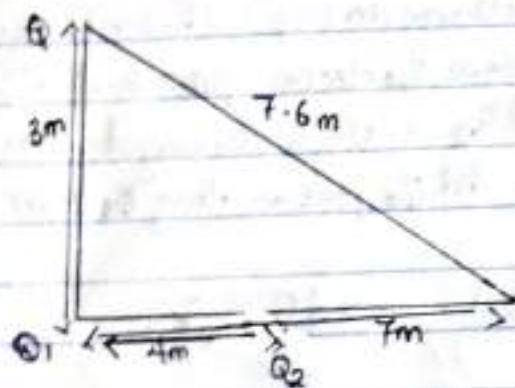
i) The net electric field at a point P on the x-axis at  $x = 7\text{m}$ .

ii) The electric field at a point Q on the y-axis at  $y = 3\text{m}$  due to the charges.

-Solution-

a) An electric field is a region of space in which an electric charge will experience an electric force while electric field intensity is the strength of electric field at any point in a space i.e. it is the force per unit charge.

2b)



where  $Q_1 = 8\text{nC}$ ,  $Q_2 = 12\text{nC}$

Recall  $E = \frac{kq}{r^2}$

$$\therefore E_1 = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2) \times (8 \times 10^{-9} \text{ C})}{(7\text{m})^2}$$

$$E_1 = 1.47 \text{ N/C}$$

$$E_2 = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2) \times (12 \times 10^{-9} \text{ C})}{(3\text{m})^2}$$

$$E_2 = 12 \text{ N/C}$$

Vector	Angle	X-component	Y-component
$E_1 = 1.47 \text{ N/C}$	$0^\circ$	$E_{1x} = 1.47 \cos 0^\circ$ $E_{1x} = 1.47 \text{ N/C}$	$E_{1y} = 1.47 \sin 0^\circ$ $E_{1y} = 0$
$E_2 = 12 \text{ N/C}$	$0^\circ$	$E_{2x} = 12 \cos 0^\circ$ $E_{2x} = 12 \text{ N/C}$	$E_{2y} = 12 \sin 0^\circ$ $E_{2y} = 0$
		$\Sigma f_x = 13.47 \text{ N/C}$	$\Sigma f_y = 0$

$$E_{\text{net}} = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$$

$$= \sqrt{(13.47)^2 + (0)^2}$$

$$E_{\text{net}} = 13.47 \text{ N/C}$$

2b.i

$$\text{Hyp}^2 = 3^2 + 4^2$$

$$\text{Hyp}^2 = 9 + 16$$

$$\text{Hyp}^2 = 25$$

$$\text{Hyp} = \sqrt{25}$$

$$\text{Hyp} = 5\text{m}$$

Electric field relative to point Q

$$E_1 = \frac{kQ_1}{r_1^2}$$

where  $r_1 = 3\text{m}$

$$E_1 = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2}$$

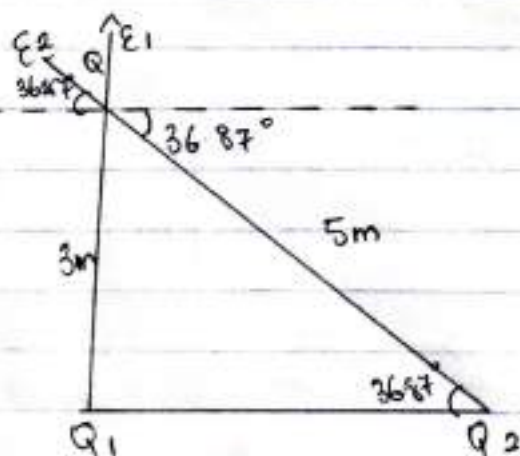
$$E_1 = \frac{72}{9} = 8\text{N/C}$$

$$E_2 = \frac{kQ_2}{r_2^2}$$

where  $r_2 = 5\text{m}$

$$E_2 = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2}$$

$$E_2 = \frac{108}{25} = 4.32\text{N/C}$$



$$\sin \theta = \frac{\text{opp}}{\text{Hyp}}, \quad \sin \theta = \frac{3}{5}, \quad \sin \theta = 0.6$$

$$\theta = \sin^{-1}(0.6)$$

$$\theta = 36.87^\circ$$

Vector	Angle	X component	Y component
$E_1 = 8 \text{ N/C}$	$90^\circ$	$8 \cos 90^\circ = 0$	$8 \sin 90^\circ = 8$
$E_2 = 4.32 \text{ N/C}$	$36.87^\circ$	$4.32 \cos 36.87^\circ = 3.46$	$4.32 \sin 36.87^\circ = 2.59$
		$\Sigma f_x = 3.46 \text{ N/C}$	$\Sigma f_y = 10.59 \text{ N/C}$

$$E = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$$

$$E = \sqrt{(3.46)^2 + (10.59)^2}$$

$$E = \sqrt{124.1197}$$

$$E = 11.14 \text{ N/C}$$

$$\tan \theta = \frac{\Sigma f_y}{\Sigma f_x} = \frac{10.59}{3.46}$$

$$\tan \theta = 3.0607$$

$$\theta = \tan^{-1}(3.0607)$$

$$\theta = 71.91^\circ$$

Question 3 State the formulation of the following identities of charges

i) Volume Charge density ii) Surface Charge density iii) Linear Charge density

b) Explain with appropriate equations the electric potential difference

c) Two point charges  $Q_1 = 10 \mu\text{C}$  and  $Q_2 = -2 \mu\text{C}$  are arranged along the x-axis at  $x = 0$  and  $x = 4\text{m}$  respectively. Find the position along the x-axis where  $V = 0$

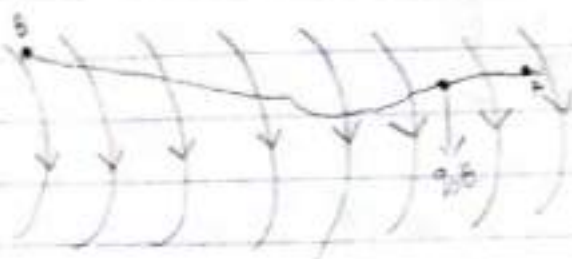
Solution

3a) i) Volume Charge density,  $\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$

ii) Surface Charge density,  $\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$

iii) Linear Charge density,  $\lambda = \frac{dQ}{dL} \rightarrow dQ = \lambda dL$

b) The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in Volt (V) or Joules per coulomb (J/C). Electric potential difference is a scalar quantity.



Consider the diagram above, suppose a test charge  $q_0$  is moved from point A to point B along an arbitrary path inside an electric field  $E$ . The electric field  $E$  exerts a force  $F = q_0 E$  on the charge as shown above. To move the test charge from A to B at constant velocity, an external force of  $F = -q_0 E$  must act on the charge. Therefore, the elemental work done  $dW$  is given as;

$$dW = F \cdot dl \dots (1)$$

But

$$F = -q_0 E \dots (2)$$

Substituting equation (2) in (1) yields

$$dW = -q_0 E dl \dots (3)$$

Then total work done in moving the test charge from A to B is;

$$W(A \rightarrow B)_{q_0} = -q_0 \int_A^B E dl \dots (4)$$

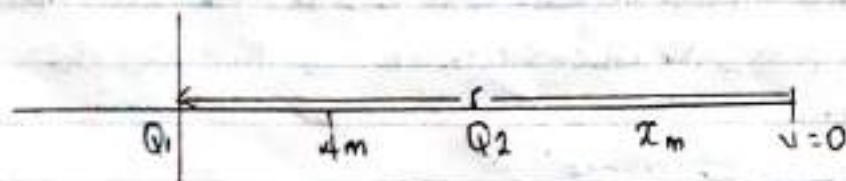
From the definition of electric potential difference, it follows that;

$$V_B - V_A = \frac{W(A \rightarrow B)_{q_0}}{q_0} \dots (5)$$

Putting equation (4) in (5) yields.

$$V_B - V_A = - \int_A^B E dl \dots (6)$$

c)



$$Q_1 = 10 \times 10^{-6} \text{ C}$$

$$Q_2 = -2 \times 10^{-6} \text{ C}$$

$$r_1 = 4 + x$$

$$r_2 = x$$

$$V_p = K \left( \frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right)$$

$$0 = 9 \times 10^9 \left[ \frac{10 \times 10^{-6}}{4+x} + \frac{-2 \times 10^{-6}}{x} \right]$$

$$0 = \left[ \frac{10 \times 10^{-6}}{4+x} - \frac{(2 \times 10^{-6})}{x} \right]$$

$$\frac{10 \times 10^{-6}}{4+x} - \frac{2 \times 10^{-6}}{x} = 0$$

$$\frac{10 \times 10^{-6} x - 2 \times 10^{-6} (4+x)}{(4+x)(x)} = 0$$

$$\frac{10 \times 10^{-6} x - (2 \times 10^{-6}) \times 4 + (2 \times 10^{-6} x)}{(4+x)(x)} = 0$$

$$\frac{10 \times 10^{-6} x - 8 \times 10^{-6} + 2 \times 10^{-6} x}{(4+x)(x)} = 0$$

$$10 \times 10^{-6} x - 8 \times 10^{-6} + 2 \times 10^{-6} x = 0 \times (4+x)(x)$$

$$10 \times 10^{-6} x + 2 \times 10^{-6} x - 8 \times 10^{-6} = 0$$

$$1.2 \times 10^{-5} x = 8 \times 10^{-6}$$

$$x = \frac{8 \times 10^{-6}}{1.2 \times 10^{-5}}$$

$$x = 0.667 \text{ m}$$

$$x \approx 1 \text{ m}$$

Since  $r_1 = 4+x$  and  $r_2 = x$

$$r_1 = 4+1 \quad r_2 = 1 \text{ m}$$

$$r_1 = 5 \text{ m}$$

$\therefore r_1 = 5 \text{ m}$  and  $r_2 = 1 \text{ m}$

Question 4a) What is magnetic flux?

b) An electron with a rest mass of  $9.11 \times 10^{-31} \text{ kg}$  moves in a circular orbit of radius  $1.4 \times 10^{-7} \text{ m}$  in a uniform magnetic field of  $3.5 \times 10^{-1} \text{ weber/meter square}$  perpendicular to the speed with which electron moves. Find the cyclotron frequency of the moving electron.

c) Discuss your answer in 4b above.

**Solution**

a) Magnetic flux can be defined as the strength of the magnetic field which can be represented by lines of forces. It is represented by the symbol  $\Phi$ .  
Mathematically given as  $\Phi = B \cdot A$

b)  $m = 9 \times 10^{-31} \text{ kg}$   $r = 1.4 \times 10^{-7} \text{ m}$   $B = 3.5 \times 10^{-1} \text{ weber/meter}^2$

Cyclotron frequency = angular speed

$$\omega = \frac{v}{r} = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$

$$\omega = 6.2222222222 \times 10^{-10} \text{ T}^{-1}$$

c) In the question we were given parameters such as

i) mass of electron =  $9.11 \times 10^{-31} \text{ kg}$

ii) A radius of  $1.4 \times 10^{-7} \text{ m}$

iii) magnetic field of  $3.5 \times 10^{-1} \text{ weber/meter square}$

and you are asked to find the cyclotron frequency which is equal or the same thing as angular speed. It is called cyclotron frequency because it is a frequency of an accelerator called cyclotron.

Recall that angular speed is given as  $\omega = \frac{v}{r} = \frac{qB}{m}$

Substituting the values given our answer is  $6.2222222222 \times 10^{-10} \text{ T}^{-1}$

So since cyclotron frequency is equal to angular speed the cyclotron frequency is equal to  $6.2222222222 \times 10^{-10} \text{ T}^{-1}$  having a unit as  $1/\text{T}$  which is equal to the unit of frequency dimensionally.

5. State the Biot-Savart Law

b) Using the Biot-Savart Law, show that the magnitude of the magnetic field of a straight current-carrying conductor is given as  $B = \frac{\mu_0 I}{2r}$

Solution

a) Biot-Savart-law states that the magnetic field is directly proportional to the product permeability of free space ( $\mu$ ), the current ( $I$ ), the change in length, the radius and inversely proportional to square of radius ( $r^2$ ).

It can be represented mathematically by  $d\vec{B} = \frac{\mu_0 I d\vec{l} \times \vec{r}}{4\pi r^2}$

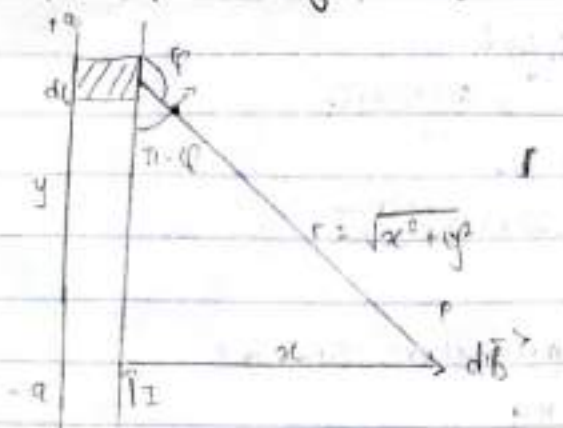
where  $\mu_0$  is a constant called permeability of free space.

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \frac{\text{m}}{\text{A}}$$

The unit of  $\vec{B}$  is weber/metre square.



5b) Magnetic field of a straight current carrying conductor.



A section of a straight current carrying conductor

Applying the Biot-Savart Law, we find the magnitude of the field  $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \theta}{r^2}$$

$$\sin(\pi - \theta) = \sin \theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{r^2}$$

From diagram,  $r^2 = x^2 + y^2$  (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{x^2 + y^2} \dots \dots \dots (1)$$

But  $\sin(\pi - \theta) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \dots \dots (2)$

Substituting (2) into (1) we have:

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2) (x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall  $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \dots \dots (3)$$

Using special integrals.

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

Equation (3) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi} \left( \frac{y}{x^2(x^2+y^2)^{1/2}} \right)_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left( \frac{2a}{x^2(x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left( \frac{2a}{(x^2+a^2)^{1/2}} \right)$$

When the length  $2a$  of the conductor is very great in comparison to its distance  $x$  from point  $P$ , we consider it infinitely long. That is, when  $a$  is much larger than  $x$

$$(x^2+a^2)^{1/2} \approx a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the  $y$ -axis. Thus at all points in a circle of radius  $r$ , around the conductor the magnitude of  $B$  is

$$B = \frac{\mu_0 I}{2\pi r} \quad \dots (4)$$

Equation (4) defines the magnitude of the magnetic field of flux density  $B$  near a long, straight current carrying conductor