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Mechatronics Engineering
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MAT 102

1 $A = 5i - 7j - 6k$, $B = j + 4k$, $C = 9i - 4j + k$
find $-8(A+B)$

Solution

$A + B = (5i - 7j - 6k) + (j + 4k) = (5i - 6j - 2k)$

$-8(A+B) = 8(5i - 6j - 2k) = -40i + 48j + 16k$

$= -40i + 48j + 16k$

$\cancel{-8(A+B)} - (C - A) = (9i - 4j + k) - (5i - 7j - 6k)$

$= (4i + 3j + 7k)$

$-8(A+B) \cdot (C - A)$

$= (-40i + 48j + 16k) \cdot (4i + 3j + 7k)$

$= -160 + 144 + 112 = 96$

$-8(A+B) \cdot (C - A) = 96$

2 $A = i + 2j - 4k$, $B = 2i - 3j + k$, $C = 4j - 3k$ - $A \times (B \times C)$

$$(B \times C) = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

$i(9-4) - j(-6-0) + k(8-0)$

$i(5) - j(-6) + k(8)$

$(B \times C) = 5i + 6j + 8k$

$$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 5 & 6 & 8 \end{vmatrix}$$

$i[16 - (-24)] - j[8 - (-20)] + k(6 - 10)$

$i(16 + 24) - j(8 + 20) + k(6 - 10)$

$i(40) - j(28) + k(-4)$

$40i - 28j - 4k$

2 Unit vector tangent to the space curve $x = -3t, y = t^2, Z = 4t^3$ at the point where $t = 1$

Solution

$r = xi + yj + zk$, $r = -3ti + t^2j + 4t^3k$, $dr/dt = -3i + (2t)j + (12t^2)k$

at $t = 1$, $dr/dt = -3t + 2(1)j + 12(1)^2 k = -3i + 2j + 12k$

$$\left|\frac{dr}{dt}\right|_{t=1} = \sqrt{(-3)^2 + (2)^2 + (12)^2} = \sqrt{9 + 4 + 144} = \sqrt{157} = 12.529$$

$$T = \frac{dr/dt}{|dr/dt|} \quad \text{Hence, } T = \frac{-3i + 2j + 12k}{12.529}$$

3) $x = 8t^2$, $y = t^2 - 4t$, $Z = t + 1$

The position vector $r = xi + yj + zk$

$r = -8t^2 i + (t^2 - 4t)j + (t+1)k$

$dr/dt = -16ti + (2t - 4)j + k$

~~Velocity~~ $\left(\frac{d^2r}{dt^2}\right) = -16i + 2j$ //

5) $R = (4\sin 3t)i + (4e^{3t})j + (7t^3)k$

$$\int R\,dt = \int_0^1 [(4\sin 3t)i + (4e^{3t})j + (7t^3)k]\,dt$$

$$= \int_0^1 [(4\sin 3t)i\,dt + (4e^{3t})j\,dt + (7t^3)k\,dt$$

$$= \left(4 \times -\tfrac{1}{3}\cos 3t\right)i \int_0^1 + \left(4 \times \tfrac{1}{3}e^{3t}\right)\int_0^1 + \left(\frac{7t^4}{4}\right)k \int_0^1$$

$$= i\left(\tfrac{4}{3}\cos 3t\right)\int_0^1 + j\left(\tfrac{4}{3}e^{3t}\right)\int_0^1 + k\left(\frac{7t^4}{4}\right)\int_0^1$$

$$= \left[\left(\tfrac{4}{3}\cos 3 - 1\right) - (0)\right] + j\left[\left(\tfrac{4}{3}e^{3-1}\right) - (0)\right] + k\left[\left(\frac{7(1)^4}{4}\right) - (0)\right]$$

$$= i\left[\tfrac{4}{3}\cos 3\right] + j\left[\tfrac{4}{3}e^3\right] + k\left[\tfrac{7}{4}\right]$$

$$= i\left[\tfrac{4}{3} \times 0.9986\right] + j\left[\tfrac{4}{3} \times 20.0855\right] + k\left[1.75\right]$$

$$= i(1.3315) + j(26.7807) + k(1.75)$$

$$= 1.3315i + 26.7807j + 1.75k$$