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DEPT: MECHATRONICS ENGINEERING

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COVID-19 HOLIDAY ASSINGMENT.

SECTION A

1a. Charging by Induction:

Electric charges can be obtained on an object without touching it, by a process called electrostatic induction.

Consider a positively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the protons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some protons move to the side of the sphere farthest away from the rod (fig. 1.3a). The region of the sphere nearest the positively charged rod has an excess of negative charge because of the migration of protons away from this location. If a grounded conducting wire is then connected to the sphere, as in (fig. 1.3b), some of the protons leave the sphere and travel to the earth. If the wire to ground is then removed (fig 1.3c), the conducting sphere is left with an excess of induced negative charge.

Finally, when the rubber rod is removed from the vicinity of the sphere (fig. 1.3d), the induced negatively charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.

Diagram:

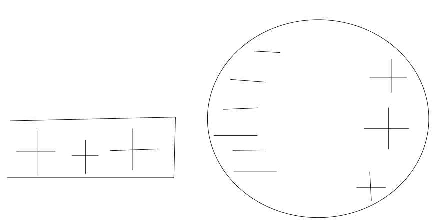


fig 1.3a

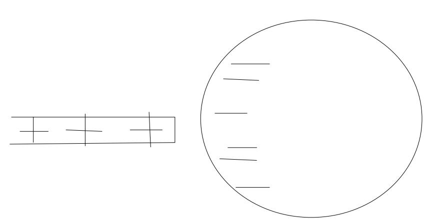


fig 1.3c

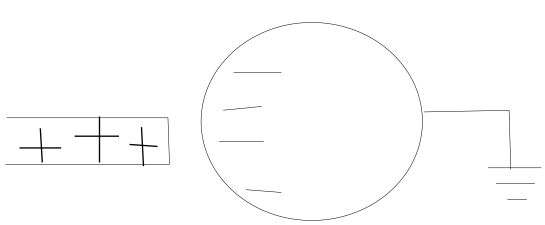


fig 1.3b

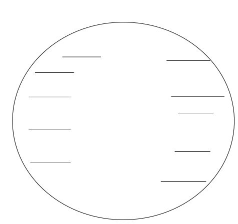


fig 1.3d

(1b) $K = 9 \times 10^9$

$q_1 + q_2 = 5\times10^{-5}\ C$

$F = 1N \quad \therefore q_1 = 5\times10^{-5} - q_2$

$d = 2m$

Calculate the Charge on each sphere

$K = \frac{1}{4\pi\varepsilon_0}$

$F = \frac{kq_1q_2}{r^2}$

$F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} \quad @ F = 1$

$1 = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \quad @ r = 2$

$4\pi\varepsilon_0(2)^2 = (5\times10^{-5} - q_2)(q_2)$

$16\pi\varepsilon_0 = 5\times10^{-5} q_2 - q_2^2$

$q_2^2 - (5\times10^{-5})q_2 + 16\pi\varepsilon_0$

$q_2 = \frac{(5\times10^{-5}) \pm \sqrt{(5\times10^{-5})^2 - 4\times1\times(16\pi\varepsilon_0)}}{2}$

$q_2 = 1.158582922\times10^{-5}$ or 3.841417058×10^{-5}

$@ q_2 = 1.158582922\times10^{-5}\ C$

$\therefore q_1 = (5 - 1.158582922)\times10^{-5} = 3.841417058\times10^{-5}$

$@ q_2 = 3.841417058\times10^{-5}\ C$

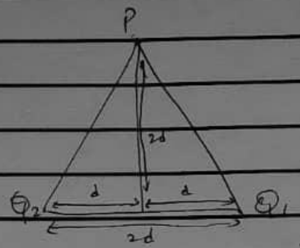
$q_1 = 1.158582922\times10^{-5}\ C$

$\therefore q_1 = 1.158582922\times10^{-5}\ C$

$q_2 = 3.841417058\times10^{-5}\ C$

1c.

1c. $Q = Q_2 = 8\mu C$ $d = 0.5m$



$x^2 = 1^2 + 0.5^2$

$x = \sqrt{1+0.25}$

$x = \sqrt{1.25} = 1.12$

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{1}{0.5} = 2$

$\theta = \tan^{-1}(2)$

$\theta = 63.4^\circ$

$r = 1.12$

$E_1 = \frac{kq_1}{r^2} = \frac{8\times10^{-6}C}{4\pi\times\varepsilon_0\times r^2}$

$= 57520.3314 \ N/C$

$E_2 = \frac{KQ_2}{r^2} = \frac{8\times10^{-6}C}{4\pi\times\varepsilon_0\times r^2} = 57520.3314 \ N/C$

$E_q = \frac{Kq}{r^2} = \frac{q}{4\pi\varepsilon_0 r^2} = 8987551788q$

E	θ	$E_x = E\cos\theta$	$E_y = E\sin\theta$
57520.33144	63.43°	-25728.31785	51445.52643
57520.33144	63.43°	25728.31785	51445.52643
8987551788q	90°	0	8987551788q
[illegible]	[illegible]	[illegible]	[illegible]
Σ		0	102891.0529 + 8987551788q

$0 = \sqrt{(102891.0529 + 8987551788q)^2 + 0^2}$

$-102891.0529 = 8987551788q$

$q = -1.144817357\times10^{-5} \ C$

$q = -11.44817357 \ \mu C$

3a.

(i) **Volume charge density, $\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$**

(ii) **Surface charge density, $\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$**

(iii) Linear charge density, $\lambda = \frac{dQ}{dL} \rightarrow dQ = \lambda dL$

3b. ELECTRIC POTENTIAL DIFFERENCE

The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in Volt (v) or Joules per Coulomb (J/C). Electric potential difference is a scalar quantity.

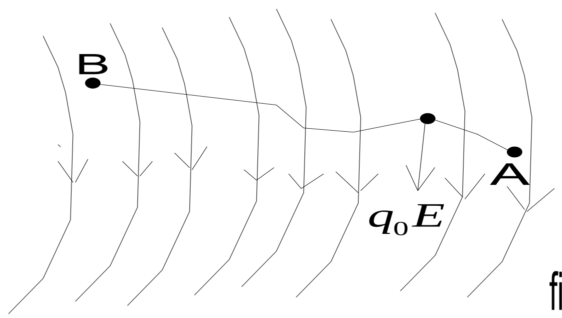


fig. 4.1

Consider the diagram above, suppose a test charge q_o is moved from point A to point B along an arbitrary path inside an electric field E. The electric field E exerts a force $F = q_oE$ on the charge as shown in fig 3.1. To move the test charge from A to B at constant velocity, an external force of $F = -q_oE$ must act on the charge. Therefore, the elemental work done dW is given as:

$$dW = F.dL \qquad \dots \qquad (1)$$

But

$$F = -q_0E \qquad \dots \qquad (2)$$

Substituting equation (2) in (1) yields

$dW = -q_0EdL \qquad \dots \qquad (3)$W-q_0EdL … (3)

Then total work done in moving the test charge from A to B is:

$$W(A \rightarrow B)_{Ag} = -q_0 \int_A^B EdL \qquad \dots \qquad (4)$$

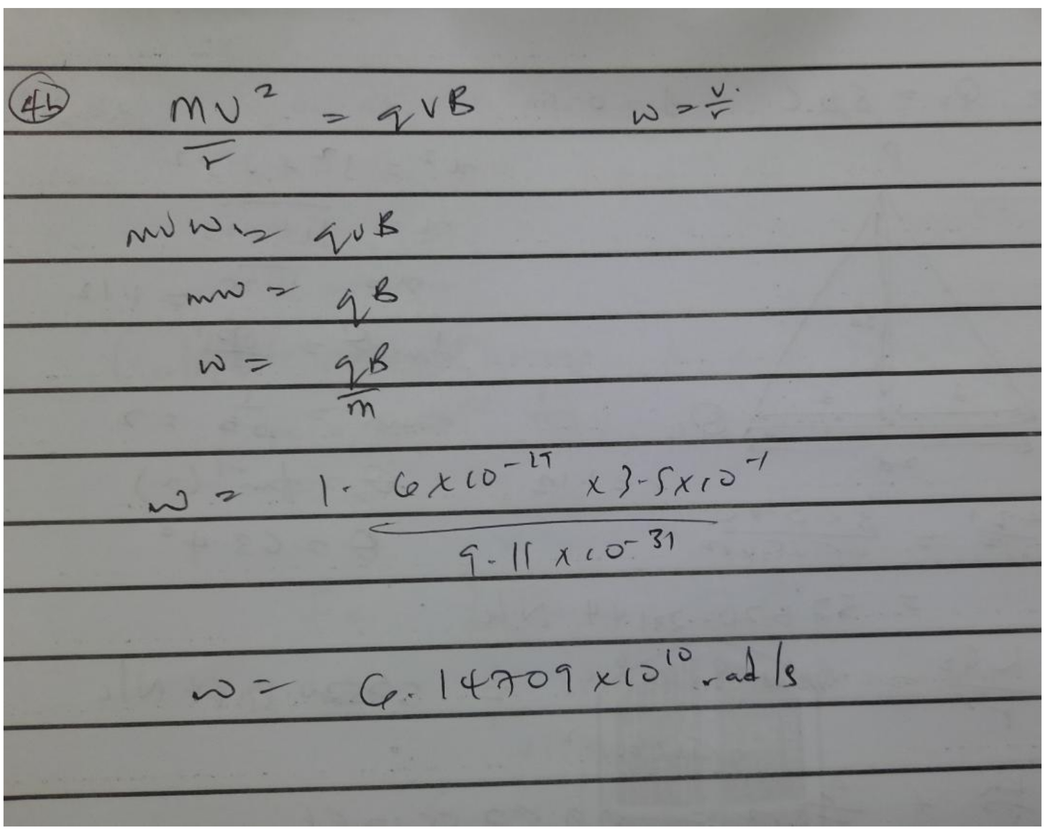
From the definition of electric potential difference, it follows that:

$V_B - V_A = \frac{W(A\rightarrow B)_{Ag}}{q_0} \qquad \dots \qquad (5)$ Putting equation (4) in (5) yields

$$V_B - V_A = -\int_A^B EdL \quad \ldots \quad (6)$$

SECTION B.

4a. magnetic flux is defined as the strength of the magnetic field which can be represented by line of forces. It is represented by the symbol Φ.mathematically given as Φ=B. d A

4b. 

(4b)

$$\frac{mv^2}{r} = qvB \qquad \omega = \frac{v}{r}$$

$$mv\omega = qvB$$

$$m\omega = qB$$

$$\omega = \frac{qB}{m}$$

$$\omega = \frac{1.6\times10^{-19} \times 3.5\times10^{-1}}{9.11\times10^{-31}}$$

$$\omega = 6.14709\times10^{10} \text{ rad/s}$$

4c. In the question we were given paramiters such as i.mass of the electron =9.11x10^{-31} kg

ii.A radius of 1.4x10^{-7}m

iii.magnetic field of 3.5x10^{-1}weber\meter square

and you are asked to find the cyclotron frequency which is equal or the same thing as angular speed.it is called cyclotron frequency because it is a frequency of an accelerator called cyclotron.

Recall that angular speed is given as ω = $\frac{v}{r}$=$\frac{qB}{m}$

Substituting we have:

ω=$\frac{v}{r}$=$\frac{qB}{m}$

$\frac{qB}{m} = \frac{1.6\times10^{-19}x3.5x10\text{^}-1}{9.11x10\text{^}-31}$=6.147091109 x 10^{10-} rad/s

SO since cyclotron frequency is equal to angular speed the cyclotron frequency is equal to =62222222222.22222^{T-1} , having a unit as 1\T which is equal to the unit of frequency dimensionally.

5b.Biot-savart law states that the magnetic field is directly proportional to the product permeability of free space(μ),the current(I),the change in length, the radius and inversely proportional to square of radius (r^2). It can be represented mathematically by

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

where μ_o is a constant called Permeability of free space.

$$\mu_o = 4\pi \times 10^{-7}\, T.\frac{m}{A}$$

The unit of $\vec{B}$ is weber\metre square

5b. Magnetic Field of a Straight Current Carrying Conductor

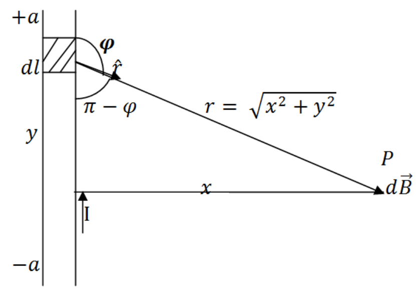


Fig 1: A section of a Straight Current Carrying Conductor

Applying the Biot-Savart law, we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a}\frac{dl \sin\varphi}{r^2}$$

$$sin(\pi - \varphi) = sin\theta$$

$$\therefore B = \frac{\mu_o I}{4\pi}\int_{-a}^{a}\frac{dl sin(\pi - \varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ $(Pythagoras\ theorem)$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a}\frac{dl sin(\pi - \varphi)}{x^2 + y^2} \quad \ldots \quad (*)$$

$$But\ \ sin(\pi - \varphi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \ldots \quad (**)$$

Substituting $(**)$ into $(*)$, we have

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a}\frac{x}{(x^2 + y^2)^{3/2}}dy$$

$$B = \frac{\mu_o I x}{4\pi}\int_{-a}^{a}\frac{1}{(x^2 + y^2)^{3/2}}dy \quad \ldots \quad (***)$$

Using special integrals:

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2}\frac{y}{(x^2 + y^2)^{1/2}}$$

Equation $(***)$ therefore becomes

$$B = \frac{\mu_o I x}{4\pi}\left[\frac{y}{x^2(x^2 + y^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_o I x}{4\pi}\left(\frac{2a}{x^2(x^2 + a^2)^{1/2}}\right)$$

$$B = \frac{\mu_o I}{4\pi x}\left(\frac{2a}{(x^2 + a^2)^{1/2}}\right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long. That is, when a is much largerthan x,

$$(x^2 + a^2)^{1/2} \cong a, as\ a \to \infty$$

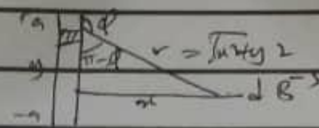
$$\therefore B = \frac{\mu_o I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y- axis. Thus, at all points in a circle of radius r, around the conductor, the magnitude of B is

$$B = \frac{\mu_o I}{2\pi r} \quad \ldots \quad (\#)$$

Equation $(\#)$ defines the magnitude of the magnetic field of flux density B near a long, straight current carrying conductor.

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$



$$\mu_0 = 4\pi \times 10^{-7}\ T\frac{m}{A}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin\theta}{r^2}$$

$$\sin(\pi - \theta) = \sin\theta$$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{dl \sin(\pi-\theta)}{r^2}$$

$r^2 = x^2+y^2$ (Pythagoras)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi-\theta)}{x^2+y^2} \quad --- (*)$$

$$\sin(\pi-\theta) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}} \quad --- (**)$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}\, dl$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}}\, dl$$

$dl = dy$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}}\, dy \quad --- (***)$$

Using special Integrals

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\frac{y}{(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

$$(x^2+a^2)^{1/2} \cong a \quad \text{as } a \to \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x} \quad --- (4)$$