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H/ENG04/038

Phd/Phd

Questions 3 & 7

3) $\frac{\partial^2 E_y}{\partial x^2} = (\mu_0 \sigma + \omega^2 \mu \epsilon) E_y$ is a familiar equation in an wave propagation in a material medium. Answer the following questions

a) Define E_y , ω , μ , σ , and ϵ Stating their units

b) If the medium is lossless, the value of σ and write the expression of the phase velocity, V_p in terms of μ and ϵ

c) If the medium is lossless and has $\mu_r = 1$ and $\epsilon_r = 1$ determine the value of V_p and the characteristic impedance Z .

d) If the wave travels in positive x -direction, and the electric field is lined up in the y -direction in what direction is the magnetic field lined up and why?

Solution:

a) E_y = electric field magnetic waves (V/m)

ω = radian frequency (rad/s)

μ = Permeability of medium (H/m)

σ = Conductivity of medium (S/m)

ϵ = Permittivity of medium (F/m)

b) When $\sigma = 0$ for a lossless medium

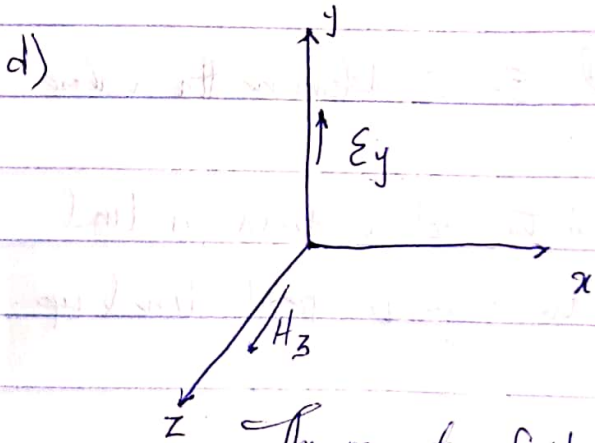
$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu \epsilon}}$$

$$c) V_p = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$$

$$\frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}} = 299792458 \text{ m/s}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}}$$

$$Z_0 = 376.73 \Omega$$



The magnetic field is lined up in the z-direction because the electric field is always perpendicular to the magnetic wave and both fields are separated at right angles to the directions of propagation of wave (i.e. x)

7) An air filled coaxial transmission line has an outer conductor inside diameter, $b = 10\text{mm}$ and an inner conductor inside diameter, $d = 3\text{mm}$ calculate the,

a) Capacitance per meter, C

b) Inductance per meter, L

c) Characteristic impedance, Z_0

d) Phase velocity, v_p of an em wave propagate through it

$$\text{Hint: } C = \frac{2\pi\epsilon_0}{\log_e b/a}; \quad L = \frac{\mu_0}{2\pi} \log_e b/a$$

Solution

$$b = 10\text{mm} = 0.01\text{m}$$

$$d = 3\text{mm} = 0.003\text{m}$$

$$\epsilon_0 = 8.854 \times 10^{-12}$$

$$\mu_0 = 4\pi \times 10^{-7}$$

$$a) \quad C = \frac{2\pi\epsilon_0}{\log_e b/a}$$

$$= \frac{2\pi \times 8.854 \times 10^{-12}}$$

$$\log_e \frac{0.01}{0.003}$$

$$C = \frac{5.563 \times 10^{-11}}{1.20397}$$

$$C = 4.6205 \times 10^{-11} \text{ F/m}$$

$$b) L = \frac{6l_0}{2\pi \log_e \frac{b}{a}}$$

$$\frac{6\pi \times 10^{-7}}{2\pi \log_e \frac{0.01}{0.003}}$$

$$2 \times 10^{-7} \times 1.20397$$

$$L = 2.4079 \times 10^{-7} \text{ Hm}^{-1}$$

$$c) Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{2.4079 \times 10^{-7}}{4.6205 \times 10^{-11}}}$$

$$Z_0 = 72.1896 \Omega$$

$$d) v_p = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{2.4079 \times 10^{-7} \times 4.6205 \times 10^{-11}}}$$

$$= 2.99791863 \cdot 2$$

$$\approx 2.998 \text{ m/s}$$

$$\approx 2.988 \text{ m/s}$$

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