a) from ; $\frac{\partial^2 E_y}{\partial x^2} = (j\omega\mu\sigma - \omega^2\mu\varepsilon)E_y$

we have $\rightarrow$ $\frac{\partial^2 E_y}{\partial x^2} = \gamma^2 E_y$.

$\gamma = \alpha + j\beta$

$E_y = E_0 e^{-\gamma x} = E_0 e^{-\alpha x} e^{-j\beta x}$

$\frac{\partial^2 E_y}{\partial x^2} = j\omega\mu\sigma E_y = \gamma^2 E_y$

or

$\gamma^2 = j\omega\mu\sigma$

$\gamma = \sqrt{j\omega\mu\sigma} = \alpha + j\beta$.

from $\sqrt{j} = \frac{1+j}{\sqrt{2}}$

$\rightarrow$ we have; $\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$

$\rightarrow \alpha \equiv \sqrt{\frac{\omega\mu\sigma}{2}}$ & $\beta \equiv \sqrt{\frac{\omega\mu\sigma}{2}}$

$\therefore E_y = E_0 e^{-\sqrt{\frac{\omega\mu\sigma}{2}}x} e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}x}$.

$\therefore$ we have; $\rightarrow E_y = E_0 e^{-\frac{x}{\delta}} e^{-j\frac{x}{\delta}}$ a..

This shows that the amplitude of the wave decreases exponentially as it penetrates a conducting medium by a factor $e^{\frac{-x}{\delta}}$.

b) Skin depth "δ" is defined as the depth of penetration of a wave inside a conductor

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

& $\omega = 2\pi f$

$$\therefore \equiv \sqrt{\frac{2}{2\pi f\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

$$\therefore \delta = \sqrt{\frac{2}{\omega\mu\sigma}} \quad \text{or} \quad \sqrt{\frac{1}{\pi f\mu\sigma}}$$

c) $f = 10\,MHz \simeq 1\times10^{7}\,Hz$

$\sigma = 5.8\times10^{7}\,S/m$, $\mu_r = 1$, $\mu_0 = 1.257\times10^{-6}$

$$\delta = \sqrt{\frac{1}{\pi f\mu\sigma}} \equiv \frac{1}{\sqrt{\pi f\mu\sigma}}$$

$$\mu = \mu_r \times \mu_0 = 1\times1.257\times10^{-6}$$
$$= 1.257\times10^{-6}$$

$$\therefore \delta = \frac{1}{\sqrt{\pi\times1\times10^{7}\times1.257\times10^{-6}\times5.8\times10^{7}}}$$

$$\delta = 2.09\times10^{-5}\,m$$

(7)

$b \simeq 10mm$, $a = 3mm$, $\epsilon_0 = 8.85 \times 10^{-12}$ F/m
$\simeq 0.01m$ $\simeq 0.003m$, $\mu_0 = 1.257 \times 10^{-6}$ H/m

(a)

Capacitance per meter, C

$$C = \frac{2\pi \epsilon_0}{\log_e b/a}$$

$$C = \frac{2\pi \times 8.85 \times 10^{-12}}{\log_e \frac{0.01}{0.003}}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12}}{\log 28.03}$$

$$C = 3.84 \times 10^{-11} \text{ F/m}$$ //

(b)

Inductance per meter, L

$$L = \frac{\mu_0}{2\pi} \log_e \frac{b}{a}$$

$$L = \frac{1.257 \times 10^{-6}}{2\pi} \log_e \frac{0.01}{0.03}$$

$$= \frac{1.257 \times 10^{-6}}{2\pi} \log 28.03$$

$$L = 2.90 \times 10^{-7} \text{ H/m}$$ //

(c)

Characteristic impedance, Z_0

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{2.90 \times 10^{-7}}{3.84 \times 10^{-11}}}$$

$$Z_0 = 86.90\ \Omega$$

(d)

Phase Velocity, V_P

$$V_P = \frac{1}{\sqrt{LC}}$$

$$V_P = \frac{1}{\sqrt{(2.90 \times 10^{-7}) \times (3.84 \times 10^{-11})}}$$

$$\cancel{V_P = 8.98 \times 10^{11}}\ V$$

$$V_P = 299664563.4$$
$$\simeq 29.10 \times 10^{7}\ V$$