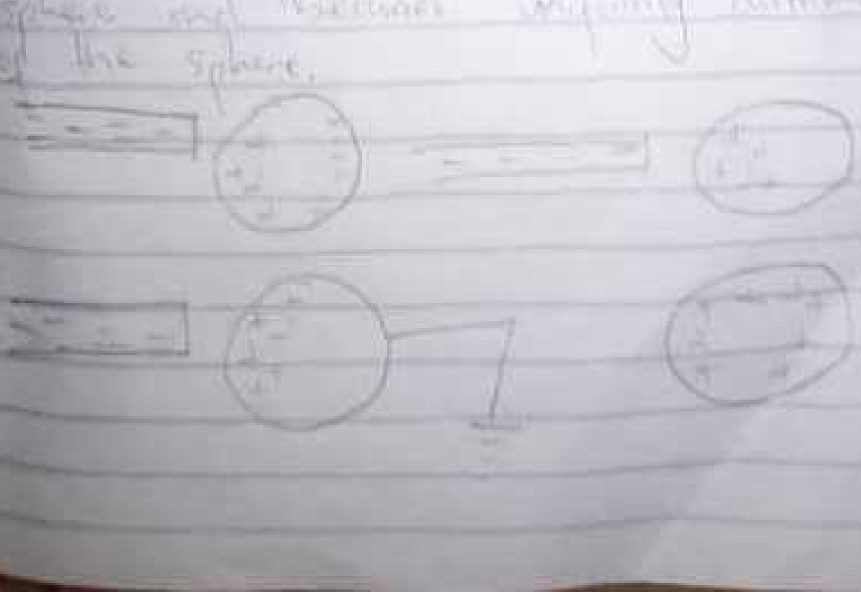
Usman Zainab Hamza
19/mhsu/1427
MBBS Maths
PHY 132

Assignment

(a) Electric charges can be obtained on an object without touching it, by a process known as electrostatic induction. Consider a negatively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground. The repulsive force between the protons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some protons move to the side of the sphere farthest away from the rod. The region of the sphere nearest the positively charged rod has ~~an~~ excess of negative charge because of the migration of protons away from this location. If a grounded conducting wire is then ~~connected~~ attached to the sphere, some of the protons leave and travel to the earth. If the wire to ground is then removed, the conducting sphere is left with an induced negative charge. Finally, when the rubber rod is removed from the vicinity of the sphere, the induced ~~negative~~ negative charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.



(b) $k = 9 \times 10^9$

$q_1 + q_2 = 5.0 \times 10^{-5} C$

$F = 1.0 N$

$d = 2 cm$

Calculate the ~~sphere~~ charge on each sphere

Recall that $k = 9 \times 10^9$

$$F = \frac{K q_1 q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 \times (q_1 q_2 \; 5.0 \times 10^{-5})}{2^2}$$

$4 = 9 \times 10^9 \times 5.0 \times 10^{-5} q_1 + 9 \times 10^9 \times 5.0 \times 10^{-5} q_2$

$4 = 4.5 \times 10^5 q_1 + 9 \times 10^9 q_2$

It is a quadratic equation

$9 \times 10^9 q_2 - 4.5 \times 10^5 q_1 + 4 = 0$

$q_1 = 0.000011 C$

$q_2 = 0.0000038 C$

$\simeq q_1 = 1.1 \times 10^{-5} C$

$\simeq q_2 = 3.8 \times 10^{-6} C$

(c) $q_1 = q_2 = 8 \mu C$

$d = 0.5 m$

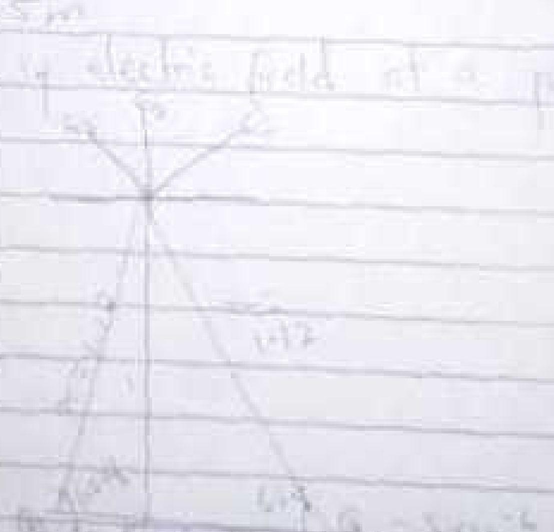
determine the electric field at a point P is 0

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = 1 \div 0.5 = 2$

$\theta = \tan^{-1}(2)$

$\theta = 63.4$



$E_1 = \frac{kq_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{1.12^2} = 5739.795918$

$E_2 = \frac{kq_2}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{1.12^2} = 5739.795918$

$E_q = \frac{kq}{r^2} = \frac{9\times10^9 \times q}{1} = 9\times10^9 q$

Vector	angle	x-comp	y-comp
$E_1 = 5739.795918$	$63.4°$	$E_1 \times \cos\theta$ $= -2570.045785$	5132.262837
$E_2 = 5739.795918$	$63.4°$	2570.045785	5132.262837
$E_q = 9\times10^9 q$	$90°$	$E_q \cos 90 = 0$	$9\times10^9 q$
		$\Sigma x = 0$	$\Sigma y = 10264.52568$

magnitude $= \sqrt{(E_x)^2 + (E_y)^2}$

$E_q = \sqrt{(0)^2 + (10264.52568)^2}$

Since $E_q = 0$

$0 = 9\times10^9 q + 10264.52658$

$q = \frac{-10264.52568}{9\times10^9}$

$q = 1.140562853 \times 10^{-6}$

$\therefore q = 11.4\mu C$

3a) Below are the formulations of the following densities in charges

(i) volume charge density: $\rho = \frac{dq}{dv} \rightarrow dq = \rho dv$

~~(ii) Surface charge density~~

(ii) Surface charge density: $\sigma = \frac{dq}{dA} \rightarrow dq = \sigma dA$

(iii) linear charge density, $\lambda = \frac{dQ}{dl} \rightarrow dQ = \lambda dl$

3b) Electric potential difference

The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge transported from one point to the other. It is measured in Volt (V) or Joules per coulomb (J/C). Electric potential difference is a scalar quantity

Considering the above diagram, suppose a test charge q_o is moved from point A to point B along an arbitrary ~~path~~ curved path inside an electric field E. The electric field E exerts a force $F = q_o E$ on the charge. To move the test charge from A to B at constant velocity, an external force of $F = -q_o E$ must act on the charge. Therefore the elemental work done dW is given as

$$dW = F \cdot dl \quad \text{——— (i)}$$

But

$$F = q_o E \quad \text{——— (ii)}$$

Substituting equation (ii) in (i) yields

$$dW = q_o E \cdot dl \quad \text{——— (iii)}$$

Then total work done in moving the test charge from A to B

$$W(A \to B)_{ext} = q_o \int_A^B E \cdot dl \quad \text{——— (iv)}$$

From the definition of electric potential difference

$$V_B - V_A = \frac{W(A \to B)_{ext}}{q_o} \quad \text{——— (v)} \quad \text{put equation (iv) into (v)}$$

$$V_B - V_A = \int_A^B E \cdot dl \quad \text{——— (vi)}$$

SECTION B

4a) Magnetic flux is defined as the strength of the magnetic field which can be represented by the flux lines. It is represented by the symbol Φ mathematically given as ∮ B . dA

4b) $m = 9 \times 10^{-31}$ kg

$r = 1.4 \times 10^{-?}$ m

$B = 3.5 \times 10^{1}$ weber/meter

cyclotron frequency = angular speed

$\omega = \frac{v}{r} = \frac{qB}{m}$

$\omega = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{1}}{9 \times 10^{-31}}$

$\omega = 6.2 \times 10^{12}$ $\omega = 6222222222\ldots$

4c) In the question above, we were given parameters such as

(i) mass of electron $= 9.11 \times 10^{-31}$ kg

(ii) Radius of $1.4 \times 10^{-?}$ m

(iii) magnetic field of 3.5×10^{1} weber/meter²

And we are asked to find the cyclotron frequency which is equal or the same thing as angular speed. It's called cyclotron frequency because it is a frequency of an accelerator called cyclotron

Recall that angular speed is given as $\omega = \frac{v}{r} = \frac{qB}{m}$

~~Substituting we have $\omega = \frac{v}{r} = \frac{qB}{m} = 1.6$~~

Substituting we have $\omega = \frac{v}{r} = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{1}}{9.11 \times 10^{-31}}$

$\frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{1}}{9.11 \times 10^{-31}} = 6222222222\ldots$

So since cyclotron frequency is equal to angular speed the cyclotron frequency is equal to 622222222…, having a unit as 1/s which is equal to the unit of frequency dimensionally

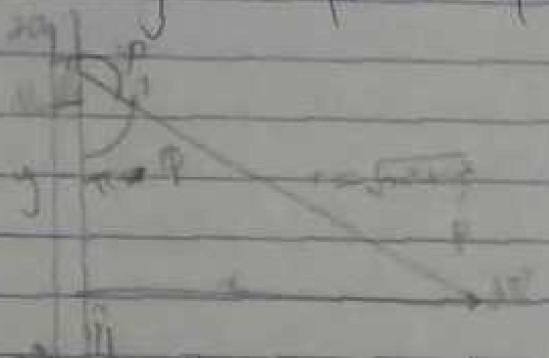
5a) Biot - Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ_0), the current (I), the change in length, the radius and inversely proportional to square of radius (r^2). It can be represented mathematically by

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

where μ_0 is a constant called permeability of free space

$\mu_0 = 4\pi \times 10^{-7}\ T.m/A$. The unit of B is weber/m²

5b) Magnetic field of a straight line carrying conductor



A section of a straight ~~line~~ current carrying conductor

Applying the Biot - Savart law, we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin\varphi}{r^2}$$

Since $(\pi - \varphi) = \sin\theta$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

From the diagram $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad \text{--- (i)}$$

But $\sin(\pi - \varphi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}}$ --- (ii)

substitute (ii) into (i), we have

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \mu_0 I \int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy \quad \text{... (iii)}$$

Using special integrals

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

Equation (iii) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2(x^2+y^2)^{1/2}} \right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2(x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P, we consider it infinitly long that is, when a is much larger than x,

$$(x^2+a^2)^{1/2} \simeq a, \text{ as } a \longrightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation we have axial symmetry about the y-axis thus at all points in a circle of radius r, around the conductor, the magnitude of B is

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{—— (v)}$$

Equation (v) defines the magnitude of the magnetic field of flux density B near a long, straight current carrying conductor