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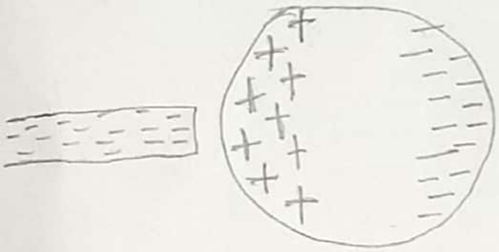
Matric No 119/MHS 11/101
Course: PHY 102

Assignment

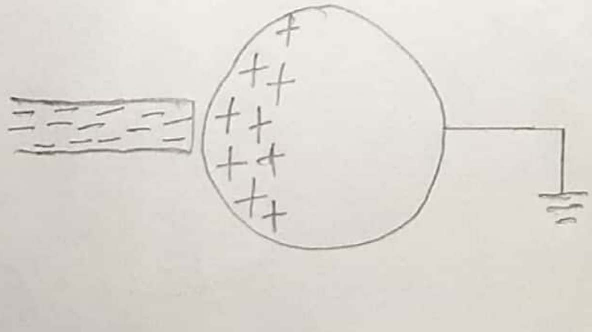
Section A

1a. Electric charge can be obtained on an object without touching it, by a process called electrostatic induction.

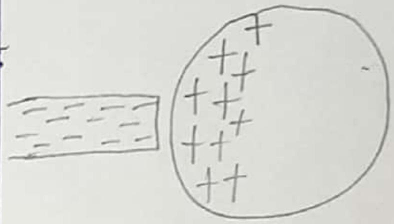
Consider a negatively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some electrons move to the side of the sphere farthest away from the rod.

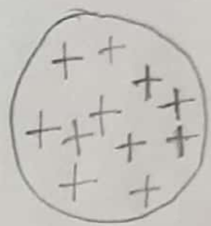


The region of the sphere nearest to the negatively charged rod has an excess of positive charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere, some of the electrons leave the sphere and travel to the earth.



If the wire to ground is then removed, the conducting sphere is left with an excess of induced positive charge. Finally, when the rubber rod is removed from the vicinity of the sphere, the induced positive charge remains on the underground sphere and becomes uniformly distributed over the surface of the sphere.





1b) $F = 1.0N$; $q_1 = ?$; $q_2 = ?$; $r = 2.0m$

$$K = \frac{1}{4\pi\varepsilon_0} = 9.0\times10^{9}\,Nm^2/C^2$$

combined charge $= 5.0\times10^{-5}C$

Using; $F = \frac{Kq_1q_2}{r^2}$

$$\frac{1}{1} = \frac{9\times10^9 \times q_1q_2}{2^2}$$

$$\frac{(9\times10^9)\times q_1q_2}{(9\times10^9)} = \frac{2^2}{(9\times10^9)}$$

$$q_1q_2 = \frac{4}{(9\times10^9)} = 4.44\times10^{-10}C^2$$

1(b) continued;

$\therefore$ combined charge $= q_1 + q_2$

$q_1 + q_2 = 5.0\times10^{-5}\,C$ ------ (i)

$q_1 q_2 = 4.44\times10^{-10}\,C^2$ ------ (ii)

From eqn (i)

$q_1 = (5.0\times10^{-5}) - q_2$

substituting q_1 into eqn (ii)

$\left((5.0\times10^{-5}) - q_2\right) q_2 = 4.44\times10^{-10}$

$5.0\times10^{-5} q_2 - q_2^2 = 4.44\times10^{-10}$

Taking the form of a quadratic equation

$q_2^2 - (5.0\times10^{-5}) q_2 + (4.44\times10^{-10}) = 0$

$(q_2 - 3.8439\times10^{-5})(q_2 - 1.14563\times10^{-5})$

$\therefore q_2 = 3.8439\times10^{-5}\,C$ or $1.14563\times10^{-5}\,C$

substitute values for q_2 into eqn (i)

For $q_2 = 3.8439\times10^{-5}\,C$

$q_1 = (5.0\times10^{-5}) - (3.8439\times10^{-5})$

$= 1.1456\times10^{-5}\,C$

For $q_2 = 1.1456\times10^{-5}\,C$

$q_1 = (5.0\times10^{-5}) - (1.1456 - 10^{-5})$

$= 3.8439\times10^{-5}\,C$

$\therefore q_1 = 1.14\times10^{-5}\,C$, $q_2 = 3.84\times10^{-5}\,C$ or vice versa.

1(c). $Q_1 = Q_2 = 8\,\mu C$

$d = 0.5m$

$x^2 = 1^2 + 0.5^2$

$x^2 = 1 + 0.25$

$\sqrt{x^2} = \sqrt{1.25}$

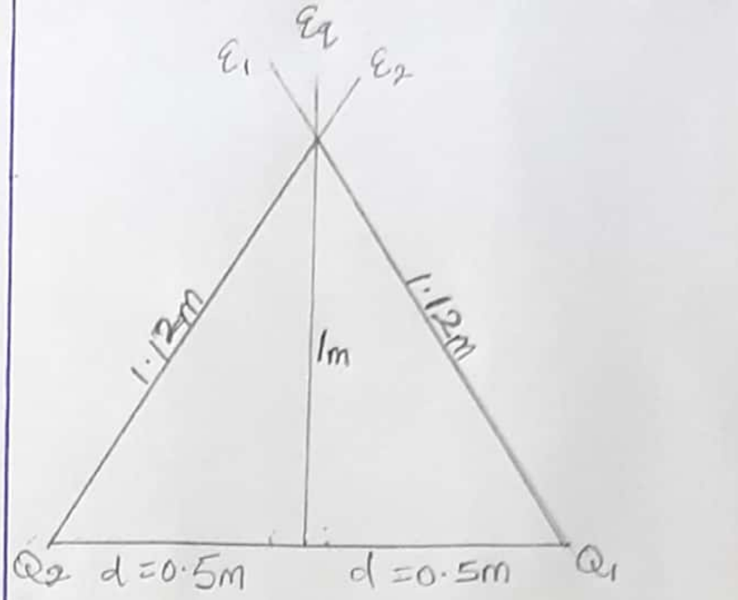
$= 1.12m$

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{1}{0.5} = 2$

$\theta = \tan^{-1}(2)$

$= 63.4^\circ$



$E_1 = \frac{Kq_1}{r_1^2} = \frac{9\times10^9\times8\times10^{-6}}{(1.12)^2} = 57397.95918$

$E_2 = \frac{Kq_2}{r_2^2} = \frac{9\times10^9\times8\times10^{-6}}{(1.12)^2} = 57397.95918$

$E_q = \frac{Kq}{r_q^2} = \frac{9\times10^9\times(q)}{1^2} = 9\times10^9 q$

Vector	Angle	x-component	y-component
$E_1 = 57397.95918$	63.4°	$E_1\times\cos\theta$ $= -25700.45785$	$E_1\times\sin\theta$ $= 51322.62839$
$E_2 = 57397.95918$	63.4°	$E_2\times\cos\theta$ $= 25700.45785$	$E_2\times\sin\theta$ $= 51322.62839$
$E_q = 9\times10^9 q$	90°	$E_q\times\cos\theta$ $= 0$	$E_q\times\sin\theta$ $= 9\times10^9 q$
		$\Sigma x = 0$	$\Sigma y = 102645.2568$

Magnitude $= \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$

$E_q = \sqrt{0^2 + (102645.2568)^2}$

$9\times10^9 q = 0 + 102645.2568$

$0 = 9\times10^9 q + 102645.2568$

$-102645.2568 = 9\times10^9 q$

making q subject of formula

$q = \frac{-102645.2568}{(9\times10^9)}$

$= -1.1405028C$

To convert to μC

$q = \frac{-1.1405028}{(10^{-6})}$

$\therefore q = -11.4\,\mu C$

3a(i) Volume charge density,

$$\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$$

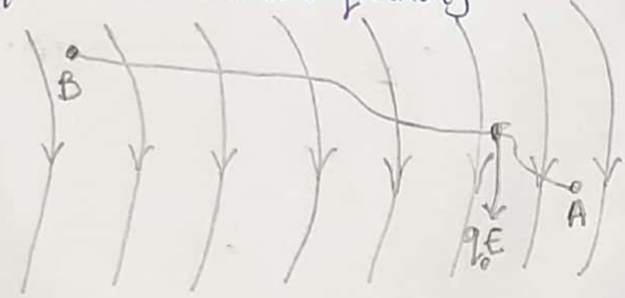
(ii). Surface charge density,

$$\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$$

(iii). Linear charge density,

$$\lambda = \frac{dQ}{dL} \rightarrow dQ = \lambda dL$$

3b. The electric potential difference between two points in an electric field can be defined as the workdone per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in Volt (V) or Joules per coulomb (J/C). Electric potential difference is a scalar quantity.



Consider the diagram above, suppose a test charge q_0 is moved from point A to Point B along an arbitrary path inside an electric field E. The electric field E exerts a force $F = q_0E$ on the charge as shown. To move the test charge from A to B at constant velocity, an external force of $F = -q_0E$ must act on the charge. Therefore the elemental workdone dW is given as:

$$dW = F.dL \text{ ------(1)}$$

But, $\quad F = -q_0E \text{ ---- (2)}$

substituting equation (2) in (1) yields

$$dW = -q_0EdL \text{ ---- (3)}$$

Then total workdone in moving the test charge from A to B is:

$$W(A \rightarrow B)_{Ag} = -q_0\int_A^B EdL \text{ ----(4)}$$

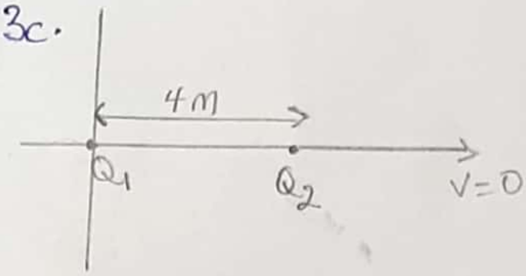
From the definition of electric potential difference, it follows that:

$$V_B - V_A = \frac{W(A \rightarrow B)_{Ag}}{q_0} \text{ ---- (5)}$$

Putting equation (4) and (5) yields

$$V_B - V_A = -\int_A^B EdL \text{ ----- (6)}$$

3c.



$Q_1 = 10\mu c = 10 \times 10^{-6} c \quad ; r_1 = 4 + x$

$Q_2 = -2\mu c = -2 \times 10^{-6} C \quad ; r_2 = x$

Recall: $V_p = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right]$

$$\frac{1}{4\pi\varepsilon_0} = K = 9 \times 10^9 Nm^2/c^2$$

$$V_p = K\left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right]$$

$$V_p = 9 \times 10^9\left[\frac{10 \times 10^{-6}}{4+x} + \frac{(-2 \times 10^{-6})}{x}\right]$$

$$0 = 9 \times 10^9\left[\frac{10 \times 10^{-6}}{4+x} - \frac{2 \times 10^{-6}}{x}\right]$$

Taking L.C.M

$$0 = 9 \times 10^9\left[\frac{10 \times 10^{-6}x - 2 \times 10^{-6}(4+x)}{x(4+x)}\right]$$

$$0 = 9 \times 10^9\left[\frac{(10 \times 10^{-6}x) - (8 \times 10^{-6}) - (2 \times 10^{-6}x)}{x(4+x)}\right]$$

$$0 = 9 \times 10^9\left[\frac{(8 \times 10^{-6}x) - (8 \times 10^{-6})}{x(4+x)}\right]$$

3c. continued;

cross multiply, we have;

$$0 \times (x(4+x)) = 9\times10^{9}((8\times10^{-6}x)-(8\times10^{-6}))$$

$$0 = 72000x - 72000$$

collecting like terms

$$\frac{\cancel{72000}\,1}{\cancel{72000}\,1} = \frac{\cancel{72000x}}{\cancel{72000}}$$

$$1 = x \quad \text{or} \quad x = 1$$

$\therefore x = r_2 = 1m$

Recall; $r_1 = 4+x$

$r_1 = 5m$

$\therefore$ Positions along the x-axis where $V=0$ is $r_1 = 5m$, $r_2 = 1m$.

Section B

4a. Magnetic flux can be defined as the strength of the magnetic field which can be represented by line of forces. It is the measure of the total magnetic field that passes through a given surface. It is represented by the symbol ϕ and the S.I unit is Weber (Wb). It can be written mathematically as;

$$\phi = B \cdot dA$$

4b. Using $\omega = \frac{qB}{M_p}$

where $q = 1.6\times10^{-19}C$; $B = 3.5\times10^{-1}$ Weber/m^2

$r = 1.4\times10^{-7}m$; $M_p = 9.11\times10^{-31}kg$

$\therefore$ Cyclotron frequency(ω) $= \frac{q_B}{M_p}$

$$= \frac{(1.6\times10^{-19})\times(3.5\times10^{-1})}{(9.11\times10^{-31})}$$

$$= 6.14709\times10^{10}$$

$$\simeq 6.15\times10^{10}\ \text{rad/s}$$

4c. In the question (4b), we were told to use values such as;

(i). mass of electron $= 9.11\times10^{-31}kg$

(ii). radius of $1.4\times10^{-7}m$

(iii). magnetic field of 3.5×10^{-1} Weber/m^2

and were then ~~called~~ asked to find the cyclotron frequency of the moving electron. This cyclotron frequency is the same thing as angular speed. This is because the charge particle circulates at this angular frequency in the type of an accelerator called Cyclotron.

Recall;

ω = angular speed = cyclotron frequency

$\omega = \frac{qB}{M_p}$. The cyclotron frequency above $= 6.15\times10^{-10}$ rad/s, also having an S.I unit of rad/s which is the unit of angular frequency dimensionally.

5a). Biot-Savart law states that the magnetic field is directly proportional to the Product permeability of free space (μ_0), the current (I), the change in length, the radius and inversely proportional to the square of radius (r^2). Mathematically, it can be written as;

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dL}\times\hat{r}}{r^2}$$

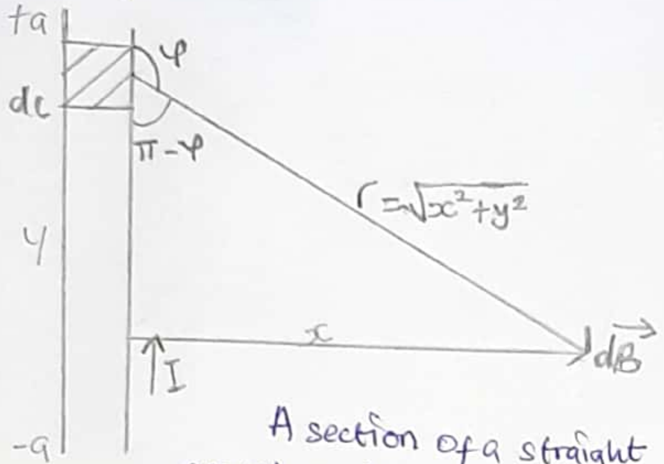
where; μ_0 is a constant called Permeability of free space.

$\mu_0 = 4\pi\times10^{-7}\ T.m/A$; r = radius

$\vec{dB}$ = magnetic field; I = steady current

$\vec{dL}$ = change in length of wire and S.I unit of $B = W/m^2$

5b. The magnetic field of a straight current carrying conductor.



A section of a straight current carrying conductor

Applying the Biot-Savart law, we find the magnitude of $\vec{dB}$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin\varphi}{r^2}$$

$$\sin(\pi-\varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin(\pi-\varphi)}{r^2}$$

From diagram, $r^2 = x^2+y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin(\pi-\varphi)}{x^2+y^2} \quad --(*)$$

But $\sin(\pi-\varphi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}} \quad --(**)$

Substituting (**) into (*), we have

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl\,\frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl\,\frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x}{(x^2+y^2)^{3/2}}\,dy$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a}\frac{1}{(x^2+y^2)^{3/2}}\,dy \quad --(***)$$

Using special integrals:

$$\int\frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\frac{y}{(x^2+y^2)^{1/2}}$$

Equation (***) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

When the length 2a of the conductor is very great in comparison to the distance x from point P, we consider it infinitely long. That is, when a is much larger than x, therefore,

$$(x^2+a^2)^{1/2} \cong a, \text{ as } a \to \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y-axis. Thus, at points in a circle of radius r, the magnitude of B around the conductor is;

$$B = \frac{\mu_0 I}{2\pi r} \quad --- (\#)$$

Equation (#) defines the magnitude of the magnetic field of flux density B near a long, straight current carrying conductor.