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19/ENG06/041
Mechanical Engineering
PHY 102

1a) Charging by Induction

Electric charges can be obtained on an object without touching it, by a process called electrostatic induction.
Consider a positively charged rubber rod brought near a neutral conducting sphere that is insulated so that there is no conducting path to the ground as shown below. The repulsive force between the protons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some electrons move to the side farthest away from the rod. The region of the sphere nearest the positively charged rubber rod has an excess of negative charge because of the migration of electrons away from its location. But if a grounded wire is then connected to the sphere, some electrons leave the sphere and travel to the earth and if the grounded wire and the rubber rod are removed from the ~~vicinity~~ vicinity of the sphere, the conducting sphere is left with an excess of induced negative charge and it is distributed uniformly over the surface of the sphere.

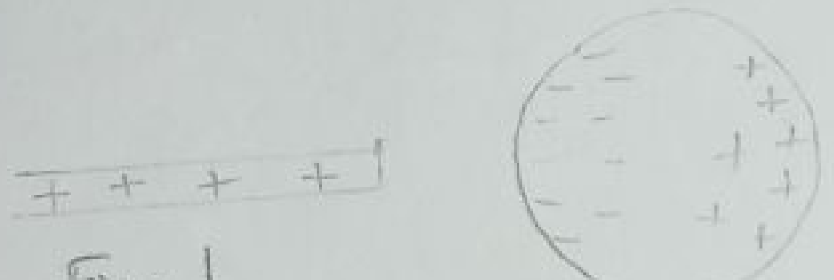
Fig. 1

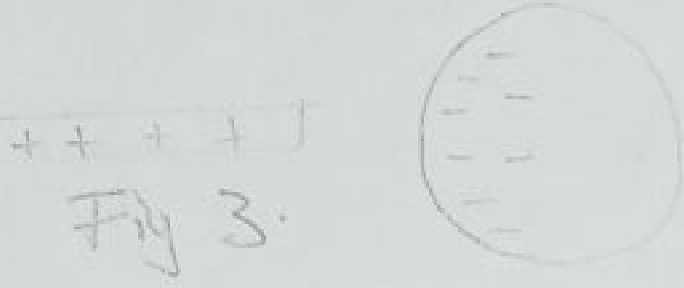
Fig 3.

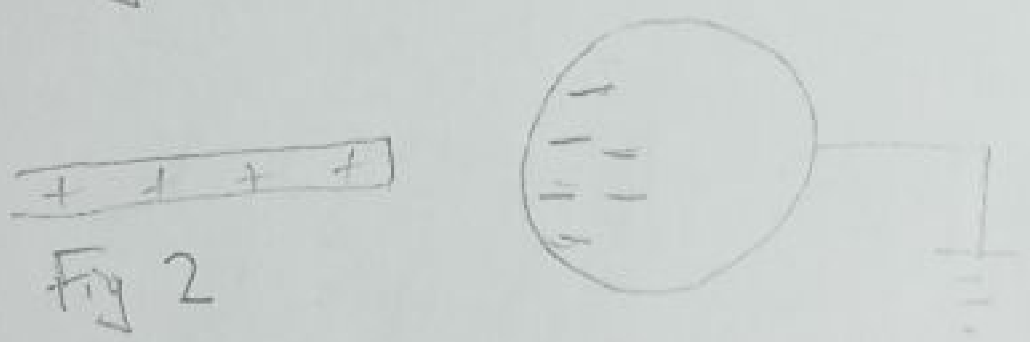
Fig 2

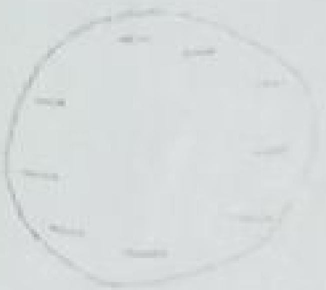
Fig. 4

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(3) $F = \frac{kq_1q_2}{r^2}$

where $F = 1N$, $r = 2m$, $q_1 + q_2 = 5.0\times10^{-5}C$,

$1.0 = \frac{9.0\times10^9 \times q_1 \times q_2}{(2)^2}$

$\frac{4}{9\times10^9} = \frac{9\times10^9 \times q_1 . q_2}{9\times10^9}$

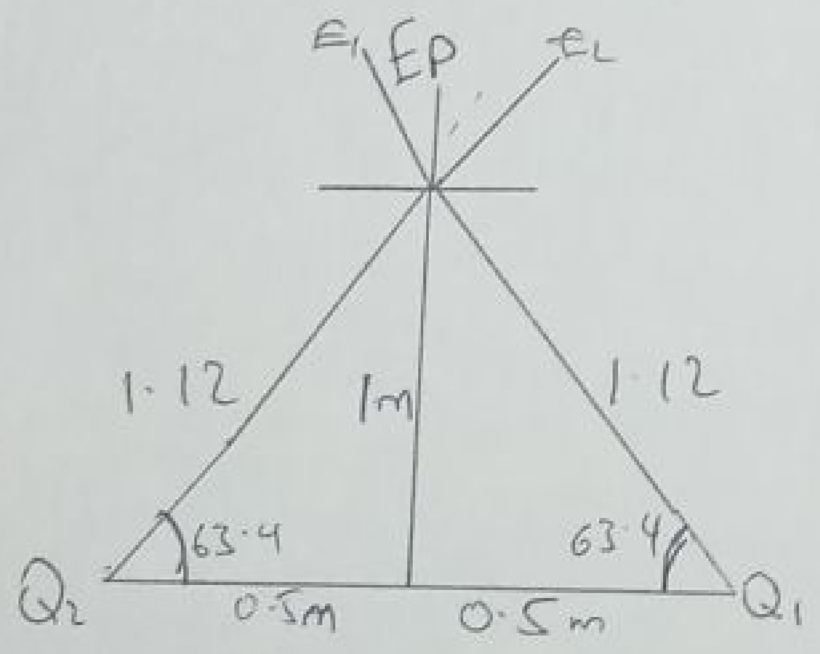
$q_1 q_2 = 4.4\times10^{-10}C^2$ ----(i)

$q_1 + q_2 = 5.0\times10^{-5}C$ ----(ii)

$q_1 = 5.0\times10^{-5}C - q_2$ ----(iii)

$(5.0\times10^{-5}C - q_2)q_2 = 4.4\times10^{-10}C^2$

1c




$x^2 = 1^2 + 0.5^2$

$x^2 = 1 + 0.25$

$x^2 = 1.25$

$x = \sqrt{1.25}$

$x = 1.12$

$Q_1 = Q_2 = 8\mu C$

$\theta = 63.4°$

$E_1 = \frac{kq_1}{r^2} = \frac{9.0\times10^9 \times 8\times10^{-6}}{(1.12)^2}$

$E_1 = 5.74\times10^4$

$E_2 = \frac{kq_2}{r^2} = \frac{9.0\times10^9 \times 8\times10^{-6}}{(1.12)^2}$

$= 5.74\times10^4$

$E_P = \frac{kP}{r^2} = \frac{9.0\times10^9 \times q}{(1)^2}$

$= 9\times10^9 q$

| Vector | $\theta$ | $x$-component | $y$-component |
|---|---|---|---|
| $E_1$ | $63.4°$ | $E_1\cos\theta =$ $5.74\times10^4\cos 63.4$ $= -2.57\times10^4$ | $E_1\sin\theta$ $5.74\times10^4\sin 63.4$ $= 5.13\times10^4$ |
| $E_2$ | $63.4$ | $2.57\times10^4$ | $5.13\times10^4$ |
| $E_p$ | $90°$ | $E_p\cos\theta$ $= 0$ | $E_p\sin\theta$ $9\times10^9 q\sin 90$ $= 9\times10^9 q$ |

$\Sigma x = 0$ , $\Sigma y = 10264.53$

$= 1.03\times10^4\ N/C$

Magnitude $= \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$

$= \cancel{\sqrt{(0)^2 + (1.03\times10^4)^2}}$

$= \cancel{1.03\times10^4}$

$0 = 9\times10^9 q + 1.03\times10^4$

Making $q$ subject of formula

$$q = \frac{-1.03\times10^4}{9\times10^9}$$

$q = -11\mu C$

2) a) i) Electric field is a region of space which an electric charge will experience an electric force.

ii) Electric field intesity, $E$, can be defined as ~~work~~ the force per unit charge. Mathematically, $E = \frac{F(N)}{q_0(C)}$, it is measured in Newton per coulomb ($N/C$).

2b) $q_1 = \cancel{4nC}\ +8nC$ , $q_2 = 12nC$

i) $E_1 = \frac{kq_1}{r^2} = \frac{9\times10^9\times(8\times10^{-9})}{7^2}$

$E_1 = \cancel{4.5N/C}\ 1.5N/C$

$E_2 = \frac{kq_2}{r^2} = \frac{9\times10^9\times(12\times10^{-9})}{3^2}$

$= 12N/C$

$E_{net} = E_1 + E_2$

$= 1.5 + 12 = 13.5N/C$

ii) Using pythagoras

$$x^2 = 4^2 + 3^2$$
$$x^2 = 16 + 9$$
$$x^2 = 25$$
$$x = \sqrt{25}$$
$$x = 5$$

$$E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times (8 \times 10^{-9})}{(3)^2}$$
$$= 8 N/C$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times (12 \times 10^{-9})}{5^2}$$
$$= 4.32 N/C$$

| Vector | $\theta$ | $x$-component | $y$-component |
|---|---|---|---|
| $E_1$ | $90^\circ$ | $E_1 \cos\theta$<br>$8 \cos 90$<br>$= 0$ | $E_1 \sin\theta$<br>$8 \sin 90$<br>$= 8 N/C$ |
| $E_2$ | $36.9$ | $E_2 \cos\theta$<br>$\cancel{4.32} \cos 36.9$<br>$= 3.45 N/C$ | $E_2 \sin\theta$<br>$4.32 \sin 36.9$<br>$2.59 N/C$ |
| | | $\Sigma x = 3.45$ | $\Sigma y = 10.59$ |

$$E = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$
$$= \sqrt{(3.45)^2 + (10.59)^2}$$
$$= \sqrt{124.05}$$
$$= 11.17 NC^{-1}$$

SECTION B

4a) Magnetic flux is defined as the stregnth of the magnetic field which can be represented by line of forces. It is represented by $\phi$. Mathematically, $\phi = B \cdot dA$

4b) $m = 9.11 \times 10^{-31}$ kg, $r = 1.4 \times 10^{-7}$ m
$B = 3.5 \times 10^{-1}$ tesla

Cyclotron frequency
$$\omega = \frac{qB}{m_e} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$
$$\omega = 6.22 \times 10^{10} T^{-1}$$

4c) In this question we were given all the parameters needed like,

i) mass of electron $= 9.11 \times 10^{-31}$ kg
ii) magnetic field of $3.5 \times 10^{-1}$ tesla
iii) A radius of $1.4 \times 10^{-7}$ m

We were asked to find the cyclotron frequency which also means angular speed.

Recall, angular speed is given as
$$\omega = \frac{v}{r} = \frac{qB}{m_e}$$

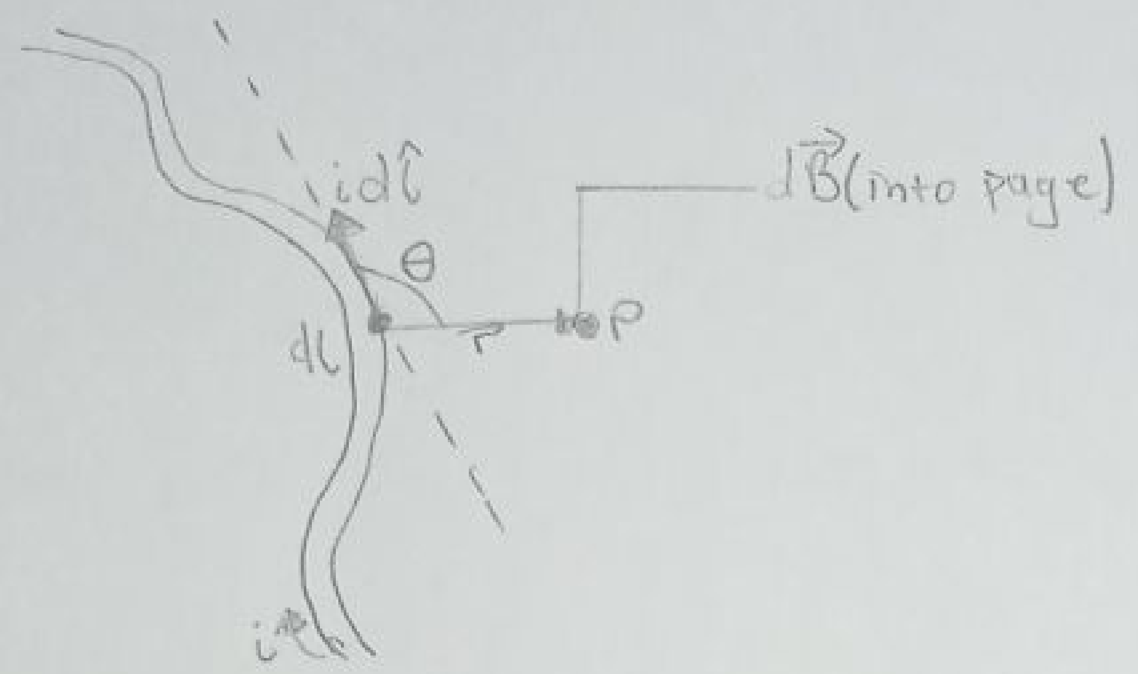
5a) Biot - Savart Law states that according to

i) The vector $d\vec{B}$ (magnetic field) is perpendicular both to $d\hat{l}$ and to the unit vector $\hat{r}$ directed from $d\vec{l}$ toward P.

ii) The magnitude of $d\vec{B}$ is inversely proportional to $r^2$, where $r$ is the distance from $d\vec{l}$ to P.

iii) The ~~magnetic~~ magnitude of $d\vec{B}$ is proportional to the current I, and to the magnitude of the length element $d\vec{l}$

iv) The magnitude of $d\vec{B}$ is proportional to $\sin\theta$, where $\theta$ is the angle between $\hat{r}$ and $dl$.

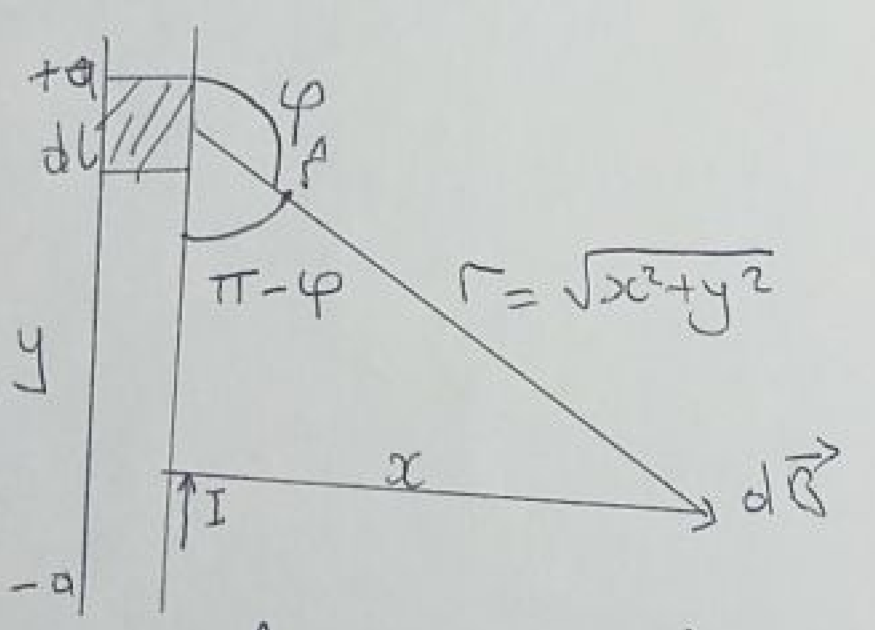



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

where $\mu_0$ is a constant called permeability of free space

$$\mu_0 = 4\pi \times 10^{-7}\ T\cdot m/A$$

5b)




A section of A straight current carrying conductor

Continuation

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl \sin\varphi}{r^2}$$

$\sin(\pi - \varphi) = \sin\theta$

$$\therefore\ B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl \sin(\pi-\varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl \sin(\pi-\varphi)}{x^2+y^2} \quad --\ (*)$$

But $\sin(\pi - \varphi) = \dfrac{x}{\sqrt{x^2+y^2}} = \dfrac{x}{(x^2+y^2)^{1/2}}$ ---(xx)

Substituting (xx) into (*), we have

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl\,\frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl\,\frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dy = dl$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x}{(x^2+y^2)^{3/2}}\,dy$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a}\frac{1}{(x^2+y^2)^{3/2}}\,dy \quad ----(***)$$

Using special integrals

$$\int\frac{dy}{(x^2+y^2)^{3/2}} = \frac{y}{x^2(x^2+y^2)^{1/2}}$$

Eqn (***) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

When the length $2a$ of the conductor is very great in comparison to its distance $x$ from point P, we consider it indefinitely long

$(x^2 + a^2)^{1/2} \approx a$, as $a \longrightarrow \infty$

$$\therefore \; B = \frac{\mu_0 I}{2\pi x}$$