

Assignment Solutions

Section A

Electric field

Electric field & electric field intensity

Electric field

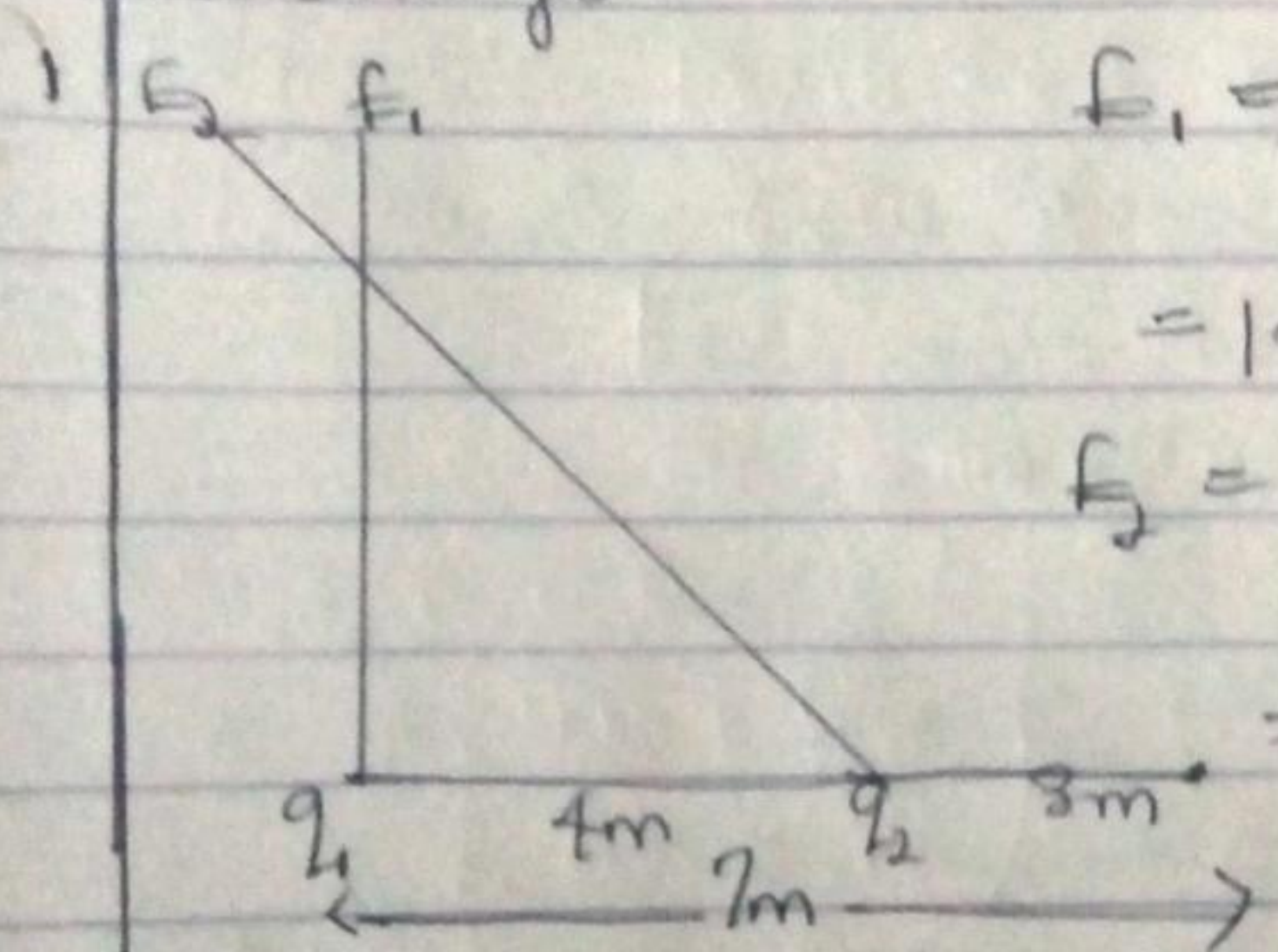
Electric field intensity

It is a region of space in which an electric charge will experience an electric force

It is a force per unit charge.

$q_1 = 8 \text{ nC}$ at origin, $q_2 = 12 \text{ nC}$ on x-axis at $x = 4 \text{ m}$

- i) net electric field at point P on the x-axis at $x = 7 \text{ m}$
- ii) electric field at a point Q on the y-axis at $y = 3 \text{ m}$ due to the charges.



$$E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2}$$

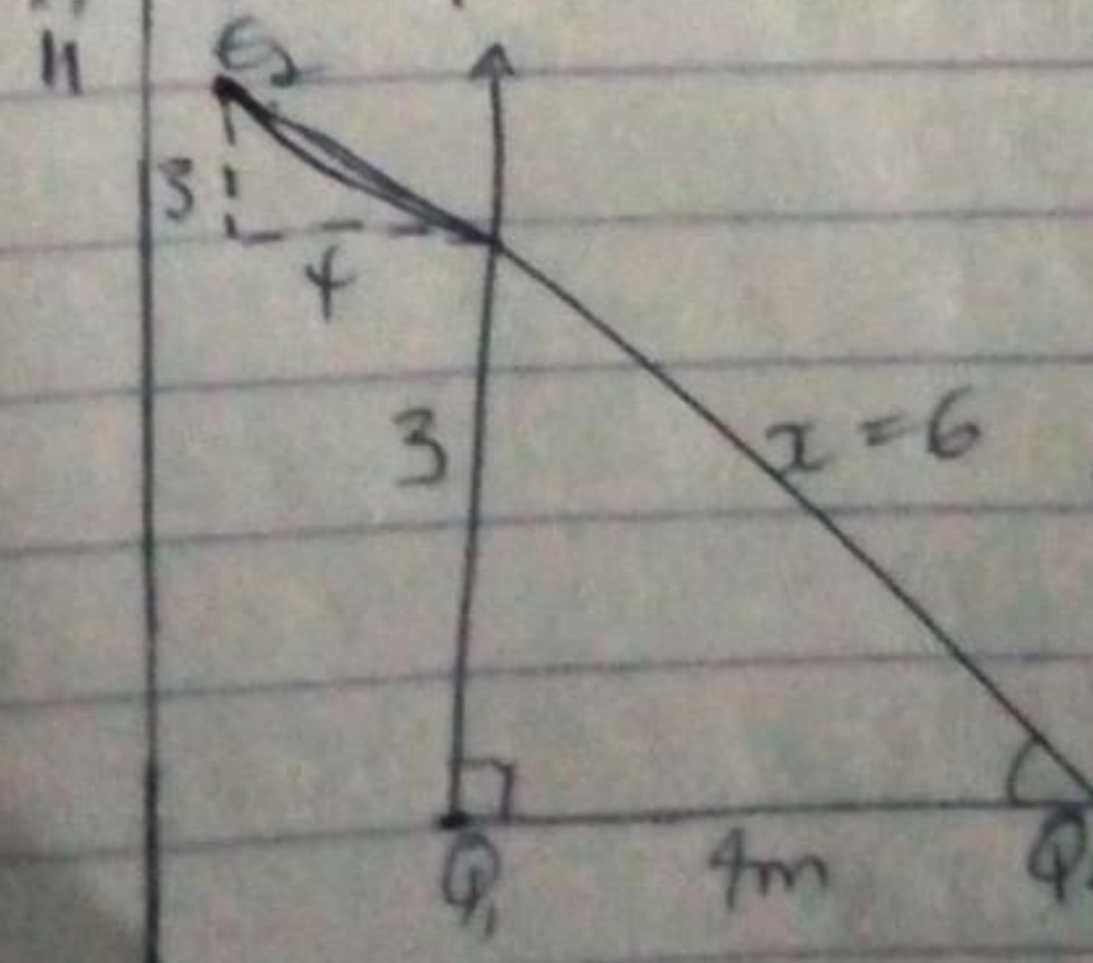
$$= 1.469 \text{ N/C} \approx 1.5 \text{ N/C}$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{7^2}$$

$$= 12 \text{ N/C}$$

$$E_{\text{net}} = E_1 + E_2 = 1.5 + 12 = 13.5 \text{ N/C}$$

ii) E at point Q on the y-axis at $y = 3 \text{ m}$ due to charge.



$$c^2 = a^2 + b^2$$

$$c^2 = 4^2 + 3^2$$

$$c = 5$$

$$E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \text{ N/C}$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32 \text{ N/C}$$

Vector	Angle	X-comp	y-comp
$F_1 = 8 \text{ N/c}$	90	0 N/c	8 N/c
$F_2 = 4.50$	36.87	-3.45 N/c	2.59 N/c
		$\Sigma F_x = -3.45 \text{ N/c}$	$\Sigma F_y = 10.59 \text{ N/c}$

$$F_{\text{net}} = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$F_{\text{net}} = 11.12 \text{ N/c}$$

3. Formulation of Identities of charges.

- i) Volume charge density $\rho = \frac{dQ}{dV} = dQ = \rho dV$
- ii) Surface charge density $\sigma = \frac{dQ}{dA} = dQ = \sigma dA$
- iii) Linear charge density $\lambda = \frac{dQ}{dL} = dQ = \lambda dL$

b) Electric Potential difference equation

=> due to a single point charge

$$V_B - V_A = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

where $Q =$ point charge

$r_B =$ distance of Q to point B

$r_A =$ distance of Q to point A

$V =$ electric potential.

=> due to several point charges.

$$V_r = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right] \text{ where}$$

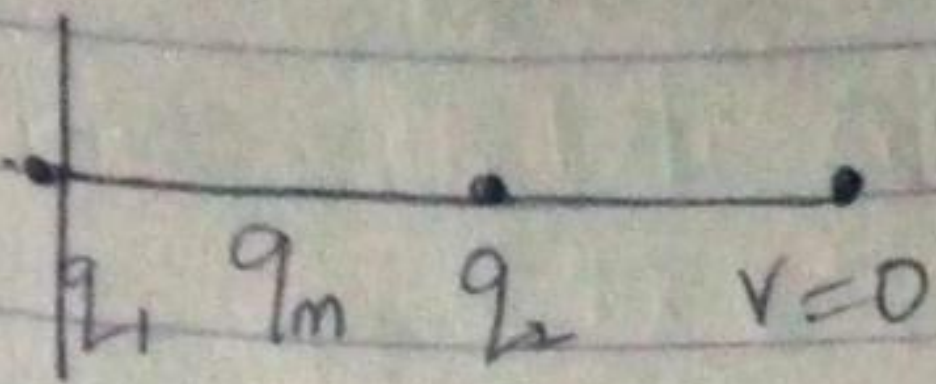
$V =$ electric potential

$Q =$ Point charge

$r =$ distance of Q

c) Point charge $Q_1 = 10 \mu\text{C}$, $Q_2 = -24 \mu\text{C}$ along x-axis $x = 2 \text{ cm} = 4 \text{ m}$ respectively
find the position along x-axis where $V = 0$

$$V_p = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right] \text{ recall } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9$$



$$V_p = k \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right]$$

$$V_p = 9 \times 10^9 \left[\frac{10 \times 10^{-6}}{4+x} + \frac{-2 \times 10^{-6}}{x} \right]$$

$$0 = 9 \times 10^9 \left[\frac{10 \times 10^{-6}}{4+x} + \frac{-2 \times 10^{-6}}{x} \right]$$

$$0 = \frac{10 \times 10^{-6}}{4+x} + \frac{-2 \times 10^{-6}}{x} = \frac{10 \times 10^{-6}}{4+x} = \frac{-2 \times 10^{-6}}{x}$$

$$10 \times 10^{-6} x = [4+x] [-2 \times 10^{-6}]$$

$$10 \times 10^{-6} x = 8 \times 10^{-6} + (-2 \times 10^{-6} x)$$

$$8 \times 10^{-6} = 10 \times 10^{-6} x - 2 \times 10^{-6} x$$

$$8 \times 10^{-6} = 8 \times 10^{-6} x$$

$$x = 1$$

\therefore Position along the x-axis is 1m

where $V = 0$

$$V = k \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right]$$

$$0 = \left[\frac{10 \times 10^{-6}}{4-x} + \frac{-2 \times 10^{-6}}{x} \right]$$

$$\frac{2 \times 10^{-6}}{x} = \frac{10 \times 10^{-6}}{4-x}$$

$$[4-x] [2 \times 10^{-6}] = 10 \times 10^{-6} x$$

$$8 \times 10^{-6} - 2 \times 10^{-6} x = 10 \times 10^{-6} x$$

$$8 \times 10^{-6} = 10 \times 10^{-6} x + 2 \times 10^{-6} x$$

$$8 \times 10^{-6} = 12 \times 10^{-6} x$$

$$x = \frac{8 \times 10^{-6}}{12 \times 10^{-6}}$$

$$x = 0.67 \text{ m}$$

\therefore Position of $V=0$ is 0.67m

SECTION B

4. Magnetic flux is defined as the strength of the magnetic field which can be represented by line of forces. It is denoted as Φ

$$\Phi = B \cdot dA$$

b. $m_e = 9.11 \times 10^{-31} \text{ kg}$, $r = 1.4 \times 10^{-7} \text{ m}$; $B = 3.5 \times 10^{-1} \text{ Wb m}^{-2}$

Cyclotron frequency = angular speed $q = 1.6 \times 10^{-19}$

$$F_B = qvB = \frac{m_e v^2}{r}$$

$$m_e v = q B r$$

$$v = \frac{q B r}{m_e} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1} \times 1.4 \times 10^{-7}}{9.11 \times 10^{-31}}$$

$$v = 8.61 \times 10^{-3} \text{ m/s}$$

$$\omega = \frac{v}{r} = \frac{q B}{m_e} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$\omega = 6.14 \times 10^{18} \text{ s}^{-1}$$

c. Mass of electron = $9.11 \times 10^{-31} \text{ kg}$

radius = $1.4 \times 10^{-7} \text{ m}$ $B = 3.5 \times 10^{-1} \text{ Wb m}^{-2}$

Cyclotron frequency = angular speed. It is referred to as cyclotron frequency because it is a frequency of an accelerator called cyclotron.

ω = angular speed

$$\omega = \frac{q B}{m_e}$$

$$\text{Cyclotron frequency} = 6.14 \times 10^{18} \text{ s}^{-1}$$

having a unit of s^{-1} which is the unit of frequency dimensionally.

5a. Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ_0)

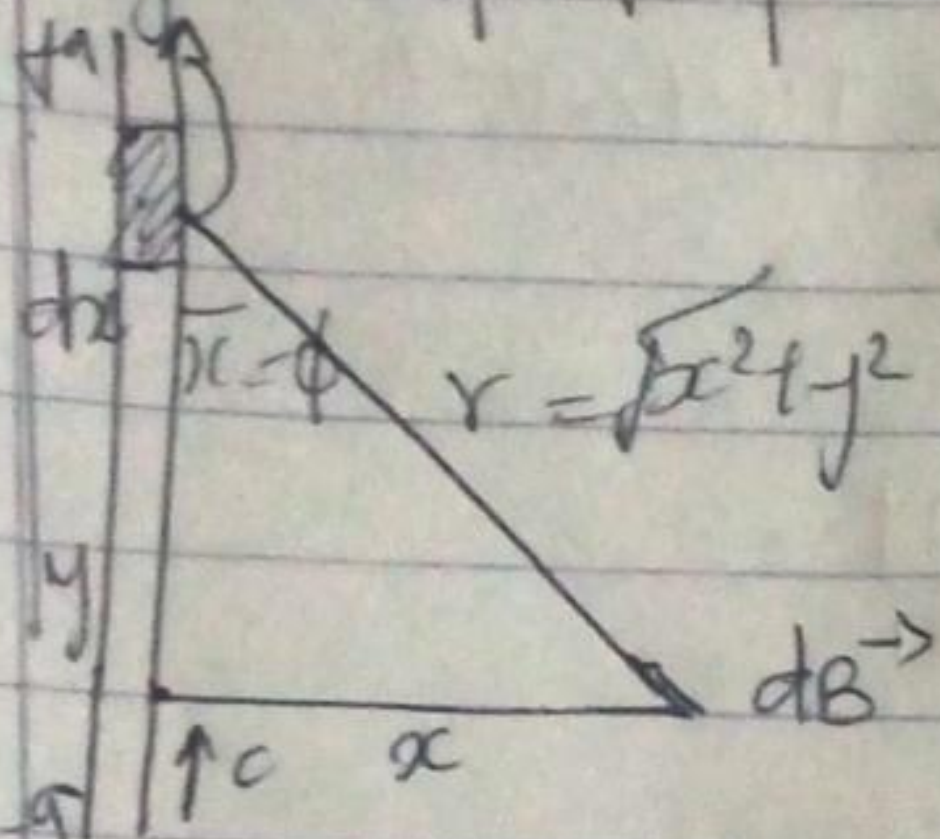
Current (I), the change in length, the radius and inversely proportional to the square of radius (r^2) mathematically.

$$dB \rightarrow = \frac{\mu_0 I dl \sin \theta}{4\pi r^2}$$

where μ_0 = Permeability of free space = $4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$
 r = radius

$dB \rightarrow$ = magnetic field, I = steady current, dl = length of wire
 μ_0 is Wb/m^2

b) Magnetic field of a straight current carrying conductor



A section of a straight current carrying conductor.

Applying Biot-Savart law, we find the magnitude of the field (dB) from the diagram.

$$B = \frac{\mu_0 I}{4\pi} \int_a^q \frac{dl \sin \theta}{r^2}$$

$$\sin(\pi - \theta) = \sin \theta$$

$$B = \frac{\mu_0 I}{4\pi} \int_a^q \frac{dl \sin(\pi - \theta)}{r^2}$$

from the diagram $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4\pi} \int_a^q \frac{dl \sin(\pi - \theta)}{x^2 + y^2} \quad \text{--- (1)}$$

$$\text{But } \sin(\pi - \theta) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \text{--- (2)}$$

Substitute the value of equ (2) into (1)

$$B = \frac{\mu_0 I}{4\pi} \int_a^q \frac{dl \cdot x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \dots \textcircled{11}$$

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi x} \left[\frac{2a}{(x^2 + a^2)^{1/2}} \right] \because (x^2 + a^2)^{1/2} = a = \textcircled{12}$$

$$B = \frac{\mu_0 I}{2\pi x} = \frac{\mu_0 I}{2\pi r}$$