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14/MHS01/342

① $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

solution.

$$\int \frac{2x}{\sqrt{4x^2-1}}\,dx$$

let $f = 4x^2 - 1$

$$\frac{df}{dx} = 8x$$

$$dx = \frac{df}{8x}$$

$$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \int \frac{\cancel{2x}}{\sqrt{f}} \cdot \frac{df}{\underset{4}{\cancel{8x}}}$$

$$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{4}\int \frac{df}{\sqrt{f}}$$

$$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{4}\int \frac{df}{f^{1/2}}$$

$$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{4}\int f^{-1/2}\,df$$

$$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{4}\left[\frac{f^{-1/2+1}}{-1/2+1}\right] + c$$

$$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{4}\left[\frac{f^{1/2}}{1/2}\right] + c$$

$$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{4}\left[f^{1/2} \div \frac{1}{2}\right] + c$$

$$= \frac{1}{4}\left[f^{1/2} \times 2\right] + c$$

$$= \frac{1}{2}\sqrt{f} + c$$

$$\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{2}\sqrt{4x^2-1} + c$$

where C is the constant of integration

② $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}$

Let $a = \sin^{-1}x$

$\frac{da}{dx} = \frac{1}{\sqrt{1-x^2}}$

$dx = \sqrt{1-x^2}\, da$

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\, dx = \int \frac{a}{\cancel{\sqrt{1-x^2}}} \cdot \cancel{\sqrt{1-x^2}}\, da$

$= \int a\, da$

$= \frac{a^{1+1}}{1+1} + c$

$\frac{a^2}{2} + c$

$\frac{(\sin^{-1}x)}{2} + c$

$\therefore \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\, dx = \frac{(\sin^{-1}x)}{2} + c$

③ $\int (\tan x)^6 \sec^2 x\, dx$

Let $v = \tan x$

$\frac{dv}{dx} = \cancel{\tan x}\, \sec^2 x$

$dx = \frac{dv}{\sec^2 x}$

$\int (\tan x)^6 \sec^2 x\, dx = \int v^6\, \cancel{\sec^2 x} \cdot \frac{dv}{\cancel{\sec^2 x}}$

$\int v^6\, dv$

$\int \frac{v^{6+1}}{6+1} = \frac{v^7}{7} + c$

$\therefore \int (\tan x)^6 \sec^2 x\, dx = \frac{(\tan x)^7}{7} + c$