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EEC 316

Q4) If the medium through which the em wave propagates is a good conductor

a) Analyse the equation in Q8 to show that the wave amplitude decreases exponentially as it penetrates the medium

b) Define the depth of penetration known as the skin depth and derive it's value in terms of the parameters of the medium and the frequency of the signal

c) Calculate the depth of penetration of the wave in a sheet of copper at a frequency of 10 MHz at which the wave amplitude decreases to 1% of it's value upon entering the sheet

Take $\sigma = 5.8 \times 10^7$ S/m, $\mu_r = 1$ for copper

SOLUTION

A) $$\frac{d^2 E_y}{dx^2} = (j\omega\mu\sigma - \omega^2\mu\varepsilon) E_y$$

$$= \Gamma^2 E_y$$

$\gamma \longrightarrow$ Propagation constant

$\gamma = \alpha + j\beta$

By suppressing the $e^{j\omega t}$

$$E_y = E_o e^{-\Gamma x} = E_o e^{-\alpha x} e^{-j\beta x} \longrightarrow \text{equ } i$$

$$\frac{d^2 E_y}{dx^2} = j\omega\mu\sigma E_y = \Gamma^2 E_y \quad \text{where } \sigma >> \omega\varepsilon$$

or $\gamma^2 = j\omega\mu\sigma$

$\gamma = \sqrt{j\omega\mu\sigma}$

$= \alpha + j\beta$

Using the fact that:

$$\sqrt{j} = \frac{1+j}{\sqrt{2}}$$

Real and Imaginary Parts

$$\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

with $a = \sqrt{\frac{\omega\mu\sigma}{2}}$

$b = \sqrt{\frac{\omega\mu\sigma}{2}}$

~~equii b~~ equation ② becomes

$$\mathcal{E}_y = \mathcal{E}_0 e^{-\sqrt{\frac{\omega\mu\sigma}{2}}x} e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}x}$$

$$\mathcal{E}_y = E_0 e^{-x/\delta} e^{-jx/\delta} \longrightarrow \text{equ (iii)}$$

where

$$\delta = \sqrt{2/\omega\mu_0} = \sqrt{2/2\pi f\mu\sigma} = \sqrt{1/\pi f\mu\sigma} \longrightarrow \text{equ (iv)}$$

Equation 3 shows that the amplitude of the wave decreases exponentially as it penetrates into a conducting medium by a factor $e^{-x/\delta}$

B) Recall:

$$\delta = \sqrt{2/\omega\mu\sigma} = \sqrt{2/2\pi f\mu\sigma}$$

$$= \sqrt{1/\pi f\mu_0\sigma}$$

$$= \frac{1}{\sqrt{\pi f\mu_0\sigma}}$$

, while δ is measured in ~~[illegible]~~ metres

μ_o = Permeability of medium Hm^{-1}
f = frequency in H_z
σ = Conductivity of medium in $\mho m^{-1}$

~~Determ~~ The values at different frequencies for copper $\mu_r = 1$, $\mu_o = 4\pi \times 10^{-7}$, $\sigma = 58M\ \mho m^{-1}$

$$\delta = \frac{6.6 \times 10^{-2}}{\sqrt{F}}$$

at specific frequencies
~~at 60Hz~~

Frequencies	Values (δ)
60 Hz	8.5×10^{-3} m
1 mHz	6.6×10^{-5} m
30 GHz	3.8×10^{-7} m
100 mHz	66×10^{-7} m
10 GHz	6.6×10^{-7} m

c) Calculate the depth of Penetration :

Preamble

$f = 10mHz$

$\sigma = 5.8 \times 10^{7}$

$\mu_r = 1$

$\mu_o = 4\pi \times 10^{-7}$

using the formula;

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\delta = \frac{1}{\sqrt{\pi \times 10 \times 10^6 \times 5.8 \times 10^7 \times 4\pi \times 10^{-7}}}$$

$\delta = 2.09 \times 10^{-5}$ m

- When the depth decreases by 1% = 10^{-2}

$e^{-x/\delta} = 10^{-2}$

$\ln(e^{-x/\delta}) = \ln(10^{-2})$

$-x/\delta = -4.61$

$-x = -4.68$

$x = 4.68$

where; $\delta = 2.09 \times 10^{-5}$

$x = 4.6 \times 2.09 \times 10^{-5}$

$x = 9.6 \times 10^{-5}$ m

∴ 1% of depth $= 9.6 \times 10^{-5}$ m

Q7) An Air-filled Coaxial transmission line has an outer conduction inside diameter, b = 10mm and an inner conductor outside diameter, a = 3 mm, Calculate the:

Solution

a) capacitance per meter, C

$C = \frac{2\pi\varepsilon_0}{\log_e b/a}$ $\quad b \rightarrow 10 \times 10^{-3}$, $a \rightarrow 3 \times 10^{-3}$

$= \frac{2 \times \pi \times 8.854 \times 10^{-12}}{\log_e 10/3 \times 10^{-3}}$

$= \frac{5.563 \times 10^{-11}}{1.2039}$

4.6208×10^{-11} Fm^{-1}

B)

$$L = \frac{\mu_0}{2\pi} \log_e {}^{b}/_{a}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \times \log_e \left(\frac{10 \times 10^{-3}}{3 \times 10^{-3}} \right)$$

$$= 2.407 \times 10^{-7} \ Hm^{-1}$$

C) $Z_0 \sqrt{L/C}$

$$= \sqrt{\frac{2.407 \times 10^{-7}}{4.6208 \times 10^{-11}}}$$

$$= 72.17 \ \Omega$$

D) $V_p = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{2.407 \times 10^{-7} \times 4.6208 \times 10^{-11}}}$$

$$= 300 \times 10^{6} \ ms^{-1}$$